MONITORING OF SERIALLY CORRELATED AND NON-NORMAL PROCESSES

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To the loving memory of my father, Alfred Osei – Aning.

To my mother, siblings and other family members.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

AEQL	:	Average Extra Quadratic Loss
ARIMA	:	Autoregressive Integrated Moving Average
ARL	:	Average Run Length
ATS	:	Average Time to Signal
CSC	:	Combined Shewhart-CUSUM
CSE	:	Combined Shewhart-EWMA
CUSUM	:	Cumulative Sum
EQL	:	Extra Quadratic Loss
EWMA	:	Exponentially Weighted Moving Average
EWRL	:	Expected Weighted Run Length
LCL	:	Lower Control Limit
Max-CUSUM	:	Maximum Cumulative Sum
Max-EWMA	:	Maximum Exponentially Weighted Moving Average
MADMAX	:	Maximum Median Absolute Deviation
MCE	:	Mixed CUSUM-EWMA
MCUSUM	:	Multivariate Cumulative Sum

MDMAX	:	Maximum average absolute deviation from Median
MEC	:	Mixed EWMA-CUSUM
MEWMA	:	Multivariate Exponentially Weighted Moving Average
PCI	:	Performance Comparison Index
QMAX	:	Maximum Interquartile range
RARL	:	Relative Average Run Length
RMAX	:	Maximum Range
SMAX	:	Maximum Standard deviation
SPC	:	Statistical Process Control
UCL	:	Upper Control Limit

ABSTRACT

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Variation is an unavoidable alteration in the conditions of a process. Statistical process control (SPC) tools such as control charts are used to monitor these variations with the aim to improve process outcome or performance. Control charts operate on the assumptions that observations are independent and normally distributed, but in reality these assumptions are usually violated.

Autocorrelation degrades the performance of control charts by producing frequent false alarms when the process is stable or lessens the sensitivity of the charts to out- of-control states. The applications of the modified and residual charts are means by which the effects due to the autocorrelation can be eliminated. We propose the Mixed EWMA-CUSUM (MEC) and Mixed CUSUM-EWMA (MCE) residual and modified charts for monitoring univariate autocorrelated data. The performance of these newly proposed charts is compared with some existing charts such as the Shewhart, CUSUM and EWMA residual and modified charts.

In the absence of normality, control charts may also malfunction and can have consequences on the error probabilities in the process. We present multivariate dispersion control charts for the efficient and robust monitoring of shifts in the covariance matrix of non-normal bivariate processes. These control charts, referred to as S*MAX*, *QMAX*,

MDMAX and *MADMAX* charts rely on dispersion estimates such as the standard deviation (*S*), interquartile range (*Q*), average absolute deviation from median (*MD*) and median absolute deviation (*MAD*) respectively. We compare the performances of these charts to the existing multivariate generalized variance $|S|$ and *RMAX* charts.

The evaluation and performance comparisons of the control charts will be based on the average run length (*ARL*) and extra quadratic loss (*EQL*) measures. Simulated and real life dataset are used to demonstrate how the charts perform. All these charts developed in this thesis will help the quality engineer to efficiently improve process performances.

خلاصة

الاسم الكامل: ريتشارد اوسي – اننج

عنوان الرسالة: مراقبة العمليّات المرتبطة ذاتيّاً وغير الطبيعيّة

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الاختلاف والتغييرأمر لا مفر منه أي عملية. تستخدم أدوات التحكم الإحصائي في العمليات مثل رسوم(مخططات) السيطرة البيانية لرصد هذه التغيرات بهدف تحسين نتائج العملية أو الأداء. تعمل رسوم المراقبة هذه على افتراض أن المعلومات مستقلة وموزعة بشكل طبيعي، ولكن الواقع أن هذه الفرضيّات غير صحيحة.

يضعف الارتباط الذاتي أداء رسوم السيطرة البيانية من خلال إنتاج الانذارات الكاذبة المتكررة في عملية مستقرة أو يقلل من حساسية الرسوم هذه عندما تكون العملية خارجة عن السيطرة. تطبيق الرسوم المعدلة والمتبقية هي وسائل لإزالة أثر الارتباط الذاتي. نقترح استخدام رسوم السيطرة البيانيّة التالية: خليط EWMA-CUSUM (MEC) وخليط CUSUM-EWMA (MCE) المتبقية والمعدلة لمراقبة المعلومات الآحاديّة المرتبطة ذاتيّا. ثم نقارن أداء هذه الرسوم المقترحة حديثا مع بعض المخططات القائمة مثل شيوارت، CUSUM وEWMA المتبقية والمعدلة.

في حالة عدم اتباع التوزيع الطبيعي، قد يكون لمخططات السيطرة البيانية المعتادة عواقب سيئةعلى احتمالات الخطأ في هذه العملية. نقدم رسوم سيطرة للتشتت في حالة المتغيرات المتعددة لرصد فعال وقوي من التحولات في مصفوفة التغاير من العمليات ذات المتغيرين غير الطبيعيّة. مخططات السيطرة هذه SMAX، QMAX، MDMAX وMADMAX تعتمد على تقديرات تشتت مثل الانحراف المعياري (S)، المدى الربعي الوسطي (Q)، متوسط الانحراف المطلق من المتوسط (MD)، الانحراف المطلق المتوسط (MAD) على التوالي. ثم نقارن أداء هذه المخططات مع مخططات التباين متعدد المتغيرات العامة القائمة ومخططات RMAX.

يستند تقييم الأداء والمقارنات من رسوم السيطرة البيانية على متوسط طول المدى (ARL) والخسارة الإضافيّة من الدرجة الثانية (EQL). وسوف نستخدم بيانات حقيقيّة وبيانات محاكاة لشرح كيفية أداء الرسوم البيانية. وجميع هذه المخططات المقترحة في هذه الأطروحة تساعد مهندس الجودة لتحسين كفاءة أداء العملية.

CHAPTER 1

INTRODUCTION

Quality is defined as “the degree to which a set of inherent characteristics fulfil a need or expectation that is stated, generally implied or obligatory” (ISO 9000: 2005). In simple terms it is a measure of the level of excellence of a product when compared to the required standards. The quality of a product can be classified as poor or good. A product can be classified as good quality if it conforms to the outlined specifications and meets the needs of customers. In the production process, the producer or manufacturer aims to produce products that meet the expected standards and are fit for use. Therefore quality assurance and improvement play an important part in the production process. To effectively improve the quality of products, statistical methods and procedures play a crucial role in the process monitoring. In quality control, we routinely monitor the process, detect any abnormalities and strive to improve the process so that products are produced with insignificant variation or no defects. In that regard, statistical process control (SPC) tools such as the histogram, check sheet, Pareto chart, cause and effect diagram, defect concentration diagram, scatter diagram and control chart may be used for process monitoring (Montgomery (2009)).

1.1. Control Chart

The Control chart is the most prominent SPC tool. The chart is used to monitor the quality characteristics of interest in the process. The control chart is a graph of the plotting statistics as the process progresses. The chart has control limits that separate common from assignable or special causes of variation. Common causes are natural, uncontrollable, random and may be attributable to noise factors. Special causes are unnatural variations that are due to controllable factors such as operator error, worn out machine part, etc. The presence of special causes may cause defects in the outputs of the process. The utilisation of control charts helps to remove special causes from the process which results in the improvement of the outputs from the process. We say a process is statistically out-of- control when observations from the process fall outside the control limits, indicating the presence of possible special causes. The expectation is that when a special cause occurs, the chart should quickly prompt the engineer of the change in the process and reveals the period the change happened.

The application of control charts in process monitoring was initially devised by Walter A. Shewhart in the 1920's. The Shewhart chart is sensitive for monitoring large shifts in the process mean. Subsequently, Page (1954) and Roberts (1959) developed the cumulative sum (CUSUM) and exponentially weighted moving average (EWMA) charts respectively for process monitoring of small shifts. Aside from their initial application in the manufacturing and industrial sector, the charts in recent times have been applied in other disciplines. These include public health care (Woodall (2006), Thor *et al*. (2007)), animal production systems (De Vries and Reneau 2010) and analytical laboratories (Abbasi 2010).

Control charts can be grouped into categories depending on the process parameter or the number of quality characteristics of interest. The parameter monitored can either be central tendency (e.g. the sample mean) or the variability of the process (e.g. standard deviation, range). In monitoring the quality characteristics, we make use of either the univariate charts (e.g. Shewhart, CUSUM and EWMA charts) or the multivariate charts (e.g. Hotelling T-square (T^2), multivariate exponentially weighted moving average (MEWMA), multivariate cumulative sum (MCUSUM) and generalized variance $|S|$). Univariate charts measure one quality variable of the process whilst multivariate charts are used to monitor more than one quality variable which are correlated with each other. Furthermore, control chart applications can be classified as Phase I or Phase II. In phase I, available historical data are gathered and used in a retrospective analysis. In this analysis process, control limits are established to ascertain if the process was in-control when the data was assembled. The primary aim is to bring the process in-control by removing out-of-control observations, compute the control limits and use them for future monitoring. The established control limits are used for continuous process monitoring in phase II to detect out-of-control points quickly.

1.2. Control Chart Assumptions

Control charts are implemented on the assumptions that observations from the process are independent and normally distributed (Montgomery (2009)). In reality this is not always the case and these fundamental assumptions are mostly violated. The charts malfunction and their performance ability is hindered under such violations.

1.2.1. Incidence of Serial Correlation

Serial correlation exists because observations from the process depend on previous observations. With the existence of serial correlation, the independence assumption cited for the use of the charts is violated. It is observed that several processes involved in refinery, smelting, wood product manufacturing operations, etc. produce correlated data (Karaoglan and Bayhan (2011)). Data from such processes are autocorrelated as a result of inertial elements and the frequent rate of sampling from the process (Harris and Ross (1991), Zhang (2000) and Montgomery (2009)). The occurrence of serial correlation in observations degrades the performance ability of the traditional control charts that are based on the independent and identically distributed (*i.i.d*) assumption. Therefore, the application of control charts on such observations eventually lead to frequent false alarms signals when the process is in-control or making the chart react slowly to the detection of out-of-control situations (Alwan and Roberts (1988), Harris and Ross (1991) and Montgomery (2009)). In literature, two methods are frequently used to eliminate or remove the impact of the serial correlation. These methods involve the use of the residual and modified charts.

Residual charts

In the procedure, the traditional control charts which satisfy the *i.i.d* assumption are applied on the residuals obtained from a time series process model that is fitted to the autocorrelated data. The use of residual charts correct the problems associated with serial correlation in the charts. The application of residual charts is presented in Chapter 2 of this thesis.

Modified charts

This procedure is implemented by applying the autocorrelated data directly on the traditional control charts, with an adjustment in the control limits of the charts to accommodate for the presence of autocorrelation. However this approach is not entirely free from time series analysis since to estimate the control limits, the extent of the autocorrelation needs to be taken into account. Chapter 3 presents application of modified charts on autocorrelated processes.

The application of these two methods on serially correlated observations during process monitoring help to improve the performance of the process. We concentrate on autocorrelation in univariate charts. In this thesis, we investigate the performance of some existing (Shewhart, CUSUM, EWMA, combined Shewhart-CUSUM (CSC) and combined Shewhart-EWMA (CSE)) residual and modified charts, and some newly proposed (Mixed EWMA-CUSUM (MEC) and Mixed CUSUM-EWMA(MCE)) residual and modified charts for the efficient monitoring of autocorrelated data. Performance measures such as average run length (*ARL*) and extra quadratic loss (*EQL*) will mainly be used to assess and compare the charts for different shifts in the mean and autocorrelation levels.

1.2.2. Optimization Design of Control Charts for Autocorrelated Processes

Every control chart has a basic design structure or mechanism which depends on certain design parameters in the formulation of the chart for process monitoring. A control chart can have either one or more design parameters which are chosen in the implementation of the charts. For example, the Shewhart chart has one parameter (L_S) whilst the CUSUM and EWMA charts have two parameters each, which are (*K*, *H*) and (L_E , λ) respectively in their design structures. The parameters $K = k\sigma$ and $H = h\sigma$ represent the reference and decision values respectively in the CUSUM chart whilst λ is the smoothing parameter in EWMA chart. The parameters in the charts are selected to achieve the desired in-control *ARL* value. Therefore, the choice of these parameters has a major influence on the performance of the chart. It is generally recommended to choose these parameters such that the charts are responsive to changes in the process with respect to shifts in the mean or variance. For a desired false alarm rate or *ARL* value, different combinations of the parameters yield the expected *ARL* value. The difficulty is to determine which specific combinations of parameters yield the best performance ability in the chart. Therefore, it is important to choose appropriate parameters to ensure better performance in the process with respect to signalling out-of-control situations when they occur.

For independent and normal data, $L_S = 3$ is usually chosen for the Shewhart chart to achieve in-control *ARL*=370. Similarly *K*=0.5 is a popular figure in the CUSUM chart since it makes the chart very sensitive to small and moderate shifts in the process (Montgomery 2009). Large values of *K* enables large shifts to go undetected and a small value increases the rate of false alarms in the chart (Karaoglan and Bayhan (2011)). The λ parameter in the EWMA chart is frequently chosen in the interval [0.05, 0.25] with $\lambda = \{0.05, 0.10, 0.20\}$ being popular parameters (Montgomery 2009). It is generally recommended to use small values of λ to detect small shifts and large values for the detection of large shifts.

In general to design a chart, one needs to determine the parameters for which the performance of the chart is optimized. These optimal parameters can be obtained through procedures that minimize control chart performance measures such as out-of-control *ARL*, *EQL*, relative average run length (*RARL*), etc. It is observed that the performance of control charts is optimized by the use of optimal parameters. In this work we optimize the designs of the modified EWMA and CUSUM charts in the presence of serial correlation with respect to the *EQL* and performance comparison index (*PCI*) values for different positive autocorrelation levels.

1.2.3. Violation of the Normality Assumption

Another important assumption cited for the application of traditional control charts is that the observations from the process must be normally distributed. Non-normal observations can be obtained from a process due to the presence of outliers, measurement error of the quality characteristic from the measuring device, few observations collected for process monitoring, etc. Other quality characteristics also follow non-normal distributions by nature. For example, it is observed that several quality characteristics in mining processes follow skewed or lognormal distributions (Samanta and Bhattacherjee (2004)). Errors may be obtained if control charts based on normal schemes are applied on non-normal data. Working with control charts for averages, Yourstone and Zimmer (1992) noted that a violation of the normality assumption can have severe consequences on the error probabilities. In the literature, several procedures are used to alleviate the problems associated with non-normal observations. An approach which can be used to deal with the problem is to increase the number of observations in the sample. The downside of this approach is that bigger sample sizes leads to a higher cost. The application of transformations such as log, square roots, etc. can be used to transform non-normal observations into normal data.

In this thesis, we concentrate on the application of bivariate charts for the simultaneous monitoring of more than one quality characteristic, which are correlated with one another. The use of univariate charts to monitor these quality characteristics independently can be misleading and yield incorrect probability assertions in relation to the type 1 error (Jackson (1959)). The implementation of multivariate charts are known to be more sensitive to special causes than univariate charts. The multivariate charts can be used to monitor shifts in the mean vector and/or covariance matrix. Several works such as Alt (1985), Alt and Smith (1988), Costa and Machado (2009, 2011), etc. have been conducted on the application of different multivariate charts on normal processes but for the purpose of this thesis, we consider the situation where observations from the process are non-normally distributed.

Amongst the collection of multivariate charts, the $|S|$ chart is widely used to monitor multivariate variability in SPC. Recently, Costa and Machado (2009, 2011) proposed the *VMAX* and *RMAX* charts which are based on the sample variance and range statistics respectively as competitors to the $|S|$ chart. In this work new dispersion charts following the *RMAX* and *VMAX* charts, referred to as *SMAX*, *QMAX*, *MDMAX* and *MADMAX* charts are presented for monitoring bivariate observations which are non-normally distributed. The performance of these charts is compared with existing charts (e.g. *RMAX* and $|S|$) for shifts in the covariance matrix using the *ARL* measure.

1.3. Performance Measures

The performance of control charts is evaluated using measures such as *ARL*, *EQL*, *PCI*, power, etc. These measures are frequently used in the comparison of control charts.

Average run length (*ARL*): *ARL* is the mean number of samples that must be plotted before a point signals an out-of-control condition (Zhang (2000)). For an in-control process, *ARL* is denoted by ARL_0 and this quantifies the false alarm rate. In process monitoring, it is expected that the *ARL* should be large when the process is in-control and small when it goes out-of-control. A chart with the least ARL_1 value is considered very sensitive to process shifts than its counterparts at a fixed ARL_0 (cf. Wu *et al*. (2009)). A Monte Carlo simulation procedure with at least 10,000 replications is used to evaluate the run length distributions of the various charts in this thesis.

Extra quadratic loss (*EQL*): *EQL* is the weighted mean *ARL* over a range of the process shift ($\delta_{\min} < \delta < \delta_{\max}$) using the square of the shift (δ^2) as the weight (cf. Abbasi *et al*. (2014)). The *EQL* is represented numerically as;

$$EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta)\, d\delta \tag{1.1}$$

A control chart with the least *EQL* value has the better performance ability compared to its counterparts over an entire shift range. The *ARL* and *EQL* performance measures (Wu *et al*. (2009), Abbasi *et al*. (2014), Ahmad *et al*. (2014)) are regularly used in process evaluation and comparison of different control charts.

Performance comparison index (*PCI*): This measure was devised by Ou *et al*. (2012). We express it as;

$$PCI = \frac{EQL}{EQL_{opt}} \tag{1.2}$$

Where EQL_{opt} is the EQL of the optimal parameters (i.e. combination of design parameters in the chart that yields the smallest EQL value). This criterion acts as a relative efficiency for the chart in relation to the parameters used. The PCI for the optimal parameters is equal to 1. The choice of parameters that yield the least PCI is considered the best.

Power: This measures the probability of detecting an out-of-control condition in the chart when it is not stable. With a fixed false alarm rate, a chart with the highest power is considered as the most efficient chart. These performance measures are usually used in the evaluation and comparison of control charts.

1.4. Outline of Thesis

This thesis is structured as follows: In Chapter 2 the MEC and MCE residual charts are proposed for the efficient monitoring of serially correlated observations that can be fitted with an autoregressive (AR) model. These charts are compared with some existing charts such as the Shewhart, CUSUM, EWMA and two combined charts of residuals. The performance comparison is done using the *ARL* and *EQL* measures.

Chapter 3 also studies the autocorrelation problem by the use of MEC and MCE modified charts. The new schemes are assessed and compared with some existing charts of observations using the *ARL* and *EQL* performance criterion for different shift and autocorrelation levels.

Chapter 4 discusses issues related to the designs and performance of the modified EWMA and CUSUM charts for autocorrelation when the level of shift is unknown or uncertain during process monitoring. An optimization procedure is used to determine optimal parameters for the charts with respect to control chart performance measures such as *EQL* and out-of-control *ARL*.

Chapter 5 presents dispersion charts for monitoring shifts in the covariance matrix of bivariate *t* and Gamma distributions. The S*MAX*, *QMAX*, *MDMAX* and *MADMAX* bivariate charts are proposed for the efficient monitoring of bivariate processes. The performance of the newly proposed charts is compared with the existing *RMAX* and $|S|$ charts using the *ARL* criterion.

Finally summaries and findings of the work are presented in Chapter 6.

CHAPTER 2

MONITORING SERIALLY CORELATED PROCESSES WITH RESIDUAL CONTROL CHARTS

2.1. Introduction

Control charts are typically used to observe irregularities that may occur in a production process so that corrective remedies are taken to curb the variation in order to produce products of good standards. Control charts usually have two limits that separate common from assignable causes of variation. Common causes in process monitoring are uncontrollable and random in nature whilst special causes on the other hand are components of the unnatural variations that are due to controllable factors such as operator error, worn out machine part, etc. Ever since Walter Andrew Shewhart proposed the first control chart in the 1920's for process monitoring, several charts have been developed over the years. The cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) charts developed by Page (1954) and Roberts (1959) respectively, are two of the most notable charts that are used in quality control to detect small shifts in the process parameters.

The usual structures of control charts are based on the assumptions that the data produced from the process are independently and normally distributed (Montgomery (2009)). The independence assumption is violated when observations from the process are serially correlated. This violation can lead to the malfunctioning of the control chart by producing high false alarm signals or slowing the detection capability of the chart when a process is unstable (Harris and Ross (1991), Zhang (2000)). Many processes in industrial and manufacturing operations produce observations which are serially correlated as a result of inertial elements and the frequent rate of sampling from the process (cf. Zhang (2000), Montgomery (2009)). Mason and Young (2002) observed that most chemical processes in manufacturing operations usually are prone to producing highly correlated data which is due to deterioration of machine part. Similarly, depletion of essential catalyst used in the process to speed up the industrial operations and contamination of equipment with chemicals also produce correlated data (cf. Mason and Yong (2002)). A simple and less complicated way of controlling the autocorrelation is by sampling less frequently from the process in order to break up the serial correlation phenomenon in the sequence of observations. The downside of this procedure is that it makes it hard to detect changes in the process since less information is available from the process. Therefore in the presence of serial correlation, the use of residual charts, modified charts (Vasilopoulos and Stamboulis (1978), Kramer and Schmid (1997), VanBrackle and Reynolds (1997)), skip sampling strategy (Costa and Castagliola (2011)), etc. are three of the elaborate remedial techniques that are adopted to deal with this phenomenon.

This chapter concentrates on the use of residual charts. In this approach, an appropriate time series model is fitted to the correlated data. The conventional charts are then applied on the residuals of the model which are expected to be uncorrelated. This procedure makes it possible to transform existing charts into residual control charts. The concept of using time series models in control charts was advanced by Alwan and Roberts (1988). Other works such as Harris and Ross(1991), Montgomery and Mastrangelo (1991), Wardel *et al*. (1992), Kramer and Schmid (1997), Yang and Makis (1997), Wieringa (1999), Lu and Reynolds (1999,2001),Zhang (2000),English *et al*. (2000), Koehler *et al*. (2001), Bisgaard and Kulahci (2005), Karaoglan and Bayhan (2011,2012), Areepong (2013) and the references therein considered serially correlated observations that can be modelled with autoregressive integrated moving average models (ARIMA) using residual control charts. A benefit of the residual chart is that it can be utilised on any form of serially correlated observations, be it stationary or otherwise (Zhang (2000), Karaoglan and Bayhan (2011)).

Over the years, a lot of emphasis has been put on the studies of the Shewhart, CUSUM and EWMA charts for location monitoring in the presence of serial correlation. Alwan and Roberts (1988) developed the Shewhart residual chart for monitoring correlated data. Harris and Ross (1991) investigated the effect of serial correlation on the performance of the EWMA and CUSUM charts. They noted that errors may be generated if the correlation behaviour of the data is not accounted for in the charts. Wardel et al. (1992) considered serial correlation that can be fitted with ARMA (1, 1) by comparing the performance of the Shewhart and EWMA charts to the common-cause and special-cause control charts. Lin (1995) considered autocorrelation that can be modelled with AR (1) and IMA (1, 1) models by applying the forecast errors from these models on the Shewhart, CUSUM, and the EWMA charts, combined Shewhart-CUSUM (CSC) and combined Shewhart-EWMA (CSE) charts. The combined charts retained the positive strengths of the Shewhart and EWMA or CUSUM charts with respect to small and large process shifts. Lu and Reynolds (1999, 2001) investigated the performance of the EWMA and CUSUM residual charts in the respective works using an AR (1) model plus a random error term.

Zhang (1998) proposed the EWMA chart for stationary process (EWMAST) and compared the performance of this chart to the Shewhart modified and residual charts for AR (1), ARMA(1,1) and AR (2) process models. Subsequently, Zhang (2000) compared the EWMAST and modified Shewhart charts to the residual Shewhart, CUSUM and EWMA charts for an AR (1) process model. He noted that the EWMAST chart performed relatively better than the other charts for small to medium shifts when the process is positively autocorrelated. Karaoglan and Bayhan (2011) focused their work on autocorrelation that can be fitted with a trend AR (1) and AR (1) process models using Shewhart type residual charts, CUSUM and EWMA residual charts. Following their previous work, they studied the performance of these charts for trend AR (1) model using vegetable oil data from industry (Karaoglan and Bayhan (2012)). They noted that for small to moderate shifts, the EWMA and CUSUM residual charts performed better than the Shewhart type charts for a moderate and strong positively correlated process.

Recently, Abbas *et al*. (2013) and Zaman *et al*. (2014) proposed the Mixed EWMA-CUSUM (MEC) and Mixed CUSUM-EWMA (MCE) charts respectively for monitoring processes for which the observations are independently and normally distributed. However, these charts malfunction in the presence of serial correlation by producing frequent false alarms. In this chapter, we propose the MEC and MCE residual charts for monitoring observations that can be modelled with an autoregressive process. We present structures of these charts with respect to the residuals obtained from the fitted model and compare the performance of the newly proposed charts to the existing charts. Comparisons among the charts are conducted for different autocorrelation and mean shift levels using individual observations.

The rest of the chapter is structured as follows: Section 2.2 describes the structure of the existing and the proposed residual chart for monitoring the autocorrelated process; the simulation process and the performance evaluation procedures are briefly discussed in Section 2.3; comparison of the charts is discussed in Section 2.4; illustrative examples to demonstrate how the charts operate are presented in Section 2.5 and finally conclusions are summarised in Section 2.6.

2.2. Residual Charts for AR (1) Processes

Serial correlation describes the relationship that exists between a variable and its past values at different time lags. In the residual chart, an appropriate time series model is fitted to the data. Subsequently, the control charts are applied on the residuals (e_t) of the model which are expected to be randomly distributed. We will restrict this work to control chart structures for serially correlated observations that can be modelled with an autoregressive AR (1) model. Several works on serial correlation in control charts have used this model (e.g. Runger and Willemain (1995), Wieringa (1999), Zhang (2000)). The general AR (1) model is represented as;

$$X_t - \mu = \phi_1 (X_{t-1} - \mu) + \varepsilon_t \tag{2.1}$$

Where X_t is the observed time series at time (t), ε_t is the white noise term, μ is the mean and ϕ_1 is the autocorrelation coefficient. ε_t is assumed to be independently and normally distributed with mean 0 and variance σ_ε^2 (i.e. $\varepsilon_t \sim N(0,\sigma_\varepsilon^2)$). To allow for the process mean shift, we include a time varying mean to equation (2.1) following Runger and Willemain (1995). This yields;

$$X_t = \mu_t + \phi_1 (X_{t-1} - \mu_{t-1}) + \varepsilon_t \tag{2.2}$$

Theoretically, in order to model assignable causes in the charts, a mean shift of magnitude (δ) is induced into equation (2.2) such that μ shifts to $\mu + \delta$. The resulting sequence of residuals (Runger and Willemain (1995)) is obtained by;

$$e_t = X_t - \widehat{X}_t \tag{2.3}$$

$$\left.\begin{array}{ll} e_t = \varepsilon_t + \delta & t = 1 \\ e_t = \varepsilon_t + (1-\phi_1)\delta & t > 1 \end{array}\right\} \tag{2.4}$$

Where $\widehat{X}_t = \mu(1-\phi_1) + \phi_1 X_{t-1}$ with the assumption that the coefficient estimates are absolutely accurate. $\widehat{X}_t$ is the predicted value of X_t. Therefore, e_t instead of the original observations (X_t) are used in the control charts.

2.2.1. Existing Control Charts for Residuals

In this section, control chart structures for the existing residual charts are briefly discussed in the presence of serial correlation. These are the Shewhart, CUSUM, EWMA, CSC and CSE residual charts.

Shewhart residual control chart

The Shewhart chart was proposed by Shewhart (1931) and it is efficient for identifying large shifts in the process when observations are independently and normally distributed. In the presence of serial correlation, the Shewhart residual chart (see Alwan and Roberts (1988), Wieringa (1999)) is used to handle the correlation. In this chart e_t in equation (2.4) is used as the plotting statistic. A process is said to be out-of-control in the chart when the plotting statistic (e_t) falls outside the following control limits.

$$UCL = \mu + L_S \sigma_e \quad \text{and} \quad LCL = \mu - L_S \sigma_e \tag{2.5}$$

Where L_S is a control chart multiplier which is used to adjust the in-control run length of the Shewhart control chart. μ and σ_e represents the in-control mean and standard deviation of the residuals from an AR (1) process.

CUSUM residual control chart

Page (1954) devised the CUSUM chart which is best noted for monitoring small changes in the mean of the process for independently and normally distributed processes. However in the presence of serial correlation this chart is transformed into the CUSUM residual chart to account for the correlation which affects the process (see Harris and Ross (1991), Wieringa (1999), Lu and Reynolds (2001)). The two sided CUSUM statistics in the residual chart are defined as;

$$\left.\begin{aligned} C_t^+ &= \max(0, e_t - \mu \ - K + C_{t-1}^+) \\ C_t^- &= \max(0, \mu - e_t - K \ + C_{t-1}^-) \end{aligned}\right\} \tag{2.6}$$

We define $K = k\sigma_e$ and $H = h\sigma_e$. Where K and H denote the reference and the decision interval values respectively. k and h are constants that are fixed to generate the in-control run length values. $k = 0.5$ is a widely used figure because it makes the CUSUM chart very sensitive to small and medium shifts (Montgomery (2009)). The chart is considered to be in an out-of-control state when either of the CUSUM statistics exceed H, i.e.

$$C_t^+ > H \quad \text{or} \quad C_t^- > H \tag{2.7}$$

EWMA residual control chart

The EWMA chart was developed by Roberts (1959) for monitoring small shifts in the process when samples from the process are independent and normally distributed. However, in the presence of serial correlation the EWMA residual chart is used (see Harris and Ross (1991), Wieringa (1999), Lu and Reynolds (1999)). For the EWMA residual chart, the statistic (W_t) is defined as:

$$W_t = \lambda e_t + (1-\lambda)W_{t-1} \tag{2.8}$$

Where λ is the smoothing parameter ($0 < \lambda \leq 1$). Small values of λ are desirable for the detection of small shifts and vice versa. The variance of W_t statistic is given as;

$$\mathrm{Var}(W_t) = \sigma_{W_t}^2 = \lim_{t\to\infty} \sigma_e^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1-(1-\lambda)^{2t}\right] = \sigma_e^2\left(\frac{\lambda}{2-\lambda}\right) \tag{2.9}$$

An out-of-control signal is generated if W_t falls outside the following control limits.

$$\mu + L_E\sigma_e\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad \text{and} \quad LCL = \mu - L_E\sigma_e\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \tag{2.10}$$

Where L_E is the control constant in the chart.

CSC residual control chart

Lucas (1982) proposed the CSC chart as an enhancement of the CUSUM chart for monitoring the location parameter of normally distributed processes. The combined chart combines the schemes of the CUSUM and Shewhart control charts. Therefore this combined chart is very sensitive to small and large shifts in the process mean. In the presence of serial correlation, the CSC residual chart was proposed by Lin (1995). A process is said to be out-of-control when the current observation (e_t) falls outside the Shewhart limits or any of the CUSUM statistics exceed the H value, i.e.

$$\left.\begin{array}{l} e_t > UCL = \mu + L_S\sigma_e \quad \text{or} \quad e_t < LCL = \mu - L_S\sigma_e \quad \text{or} \\ C_t^+ > H \quad \text{or} \quad C_t^- > H \end{array}\right\} \tag{2.11}$$

CSE residual control chart

The CSE chart was also developed as an enhancement chart of the EWMA chart. It was proposed by Lucas and Saccucci (1990) for monitoring the process mean of normally distributed data and it operates by adding the Shewhart scheme to the EWMA control chart. This combined chart is also very sensitive to small and large shifts in the process mean. The CSE residual chart is used for location monitoring when observations from the process are correlated (Lin (1995)). The CSE residual chart is considered out-of-control if either the EWMA statistic (W_t) or the current observation (e_t) falls outside their respective control limits, i.e.

$$
\left.\begin{array}{l}
e_t > UCL = \mu + L_S\sigma_e \quad \text{or} \quad e_t < LCL = \mu - L_S\sigma_e \quad \text{or} \\
W_t > UCL = \mu \ + L_E\sigma_e\sqrt{\left(\frac{\lambda}{2-\lambda}\right)} \quad \text{or} \quad W_t < LCL = \mu \ - L_E\sigma_e\sqrt{\left(\frac{\lambda}{2-\lambda}\right)}
\end{array}\right\} \qquad (2.12)
$$

2.2.2. Proposed Control Charts for Residuals

In this section, we present the control chart structures for the newly proposed MEC and MCE residual charts for monitoring autocorrelated processes.

MEC residual control chart

The MEC chart was proposed by Abbas *et al*. (2013) for monitoring independent and normally distributed processes. The MEC integrates the EWMA statistic (W_t) into the CUSUM statistic. This chart is effective for monitoring small shifts in the mean of a process. In this study, we propose the MEC residual chart as a chart for monitoring serial correlated observations. This residual chart is represented as;

$$
\left.\begin{array}{l}
MEC_t^+ = \max(0, W_t - \mu - K_t + MEC_{t-1}^+) \\
MEC_t^- = \max(0, \mu - W_t - K_t + MEC_{t-1}^-)
\end{array}\right\} \qquad (2.13)
$$

We define $K_t = k^*\sigma_W$ and $H_t = h^*\sigma_W$. Where K_t and H_t denoted the reference and decision interval values respectively. k^* and h^* are constants that are fixed to generate the in-control run length values. This chart is said to be out-of-control when either of the MEC statistics exceed H_t value, i.e.

$$
MEC_t^+ > H_t \quad \text{or} \quad MEC_t^- > H_t \qquad (2.14)
$$

MCE residual control chart

The MCE chart was proposed by Zaman *et al*. (2014) for monitoring of independent and normal processes. This chart is formulated by incorporating the CUSUM statistics into the EWMA statistic. It retains the good characteristics of both CUSUM and EWMA charts and eliminates the inability of both charts to detect large shifts when used in isolation. This chart is noted to perform relatively better than the Shewhart chart for large shifts. To eliminate the usual problems associated with control chart utilisation in the presence of serial correlation, this MCE chart is transformed into the MCE residual chart to improve process performance. The proposed transformed chart is defined as;

$$\left.\begin{aligned} MCE_t^+ &= \lambda C_t^+ + (1-\lambda)\, MCE_{t-1}^+ \\ MCE_t^- &= \lambda C_t^- + (1-\lambda)\, MCE_{t-1}^- \end{aligned}\right\} \tag{2.15}$$

Where C_t^+ and C_t^- represent the CUSUM statistics given in equation (2.6). The starting values of the MCE statistics in equation (2.15) are set equal to the grand mean of C_t^+ and C_t^- respectively. That is $MCE_0^+ = MCE_0^- = \mu_C$

The mean and variance of the MCE statistics are time varying and are obtained through a simulated procedure. They are represented as;

$$\text{mean}(MCE_t^+) = \text{mean}(MCE_t^-) = \mu_{MCE_t} \tag{2.16}$$

$$\text{Var}(MCE_t^+) = \text{Var}(MCE_t^-) = \sigma_{MCE_t}^2 \tag{2.17}$$

However as $t \to \infty$, the target mean of the CUSUM statistics become equal to the target mean of the MCE statistics. That is $\mu_{MCE} = \mu_C$. An out-of-control signal is generated if either MCE_t^+ or MCE_t^- fall outside the control limit, i.e.

$$MCE_t^+ > UCL_t = \mu_{MCE_t} + L_M \sigma_{MCE_t} \quad \text{or} \quad MCE_t^- > UCL_t = \mu_{MCE_t} + L_M \sigma_{MCE_t} \qquad (2.18)$$

Where L_M is the control constant in the chart.

2.3. Performance Evaluation

The performance evaluation of all the residual charts discussed in Section 2.2 will be done using the *ARL* and *EQL* performance criteria. These performance criteria are discussed in Section 1.3. To determine these criteria, we considered $\delta = 0, 0.50, 1, 1.50, 2, 2.5, 3, 3.5, 4$ as the values of the mean shift and $\phi = 0, \pm 0.25, \pm 0.50, \pm 0.75, \pm 0.90$ as the values of the autocorrelation coefficient. We categorise the mean shift as small ($\delta < 1$), medium ($1 \le \delta < 3$) and large ($\delta \ge 3$). For each level of mean shift and autocorrelation coefficient considered, 10000 simulations are done to obtain the *ARL* values. The approximate value of the *EQL* will be determined numerically by using the Trapezium rule.

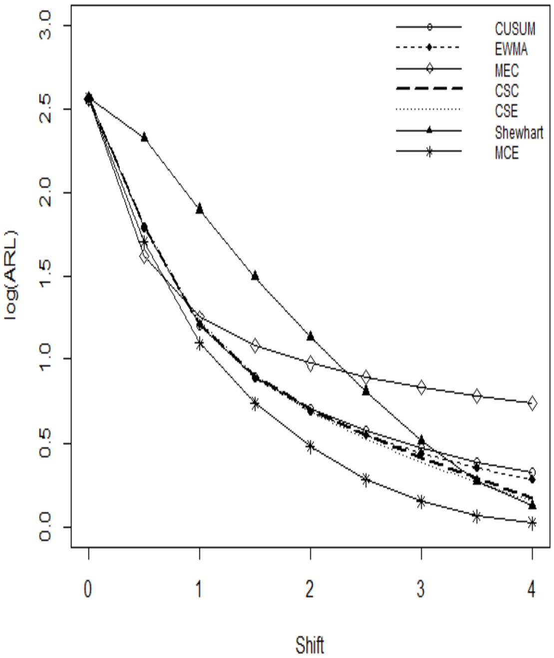
All the design parameters in the charts are adjusted in order to have in-control *ARL* of 370. We use ($L_S = 3$),($k = 0.5, h = 4.77$),($L_E = 2.86$, $\lambda = 0.2$),($L_S = 3.5$, $k = 0.5, h = 4.914$), ($L_S = 3.5$, $L_E = 2.91$, $\lambda = 0.2$), ($k = 0.5, h = 21.28, \lambda = 0.2$) and ($k = 0.5$, $\lambda = 0.2, L_M = 4.18$) design parameters in the Shewhart, CUSUM, EWMA, CSC, CSE, MEC and MCE charts respectively. The specific value of $L_s = 3.5$ is used in the combined charts because Lucas (1982) and Lucas and Saccucci (1990) recommend a value greater than the Shewhart limit of 3 (i.e. 3.5 or 4), since a larger value has a lesser effect on the in-control *ARL*. The *ARL* and *EQL* values for the residual charts are displayed in Tables 2.1 and 2.2 for an AR (1) process with $\phi \geq 0$ and $\phi < 0$ respectively. In Table 2.3, we computed the *ARL* and *EQL* values for the charts when $\phi = 0.25$ for $-\delta$. It is observed that the *ARL* and *EQL* values for both δ and $-\delta$ are similar for ϕ (see Tables 2.1 and 2.3 when $\phi = 0.25$). Figures 2.1 and 2.2 present graphical comparison of the charts for different autocorrelation and mean shift levels. In this work, we have replicated the *ARL* results of the EWMA and CUSUM charts from the work of Zhang (2000) to ensure the validity of the simulation procedures used.

Table 2.1: *ARL* and *EQL* values for the residual charts at different shifts (δ) when $\phi \geq 0$

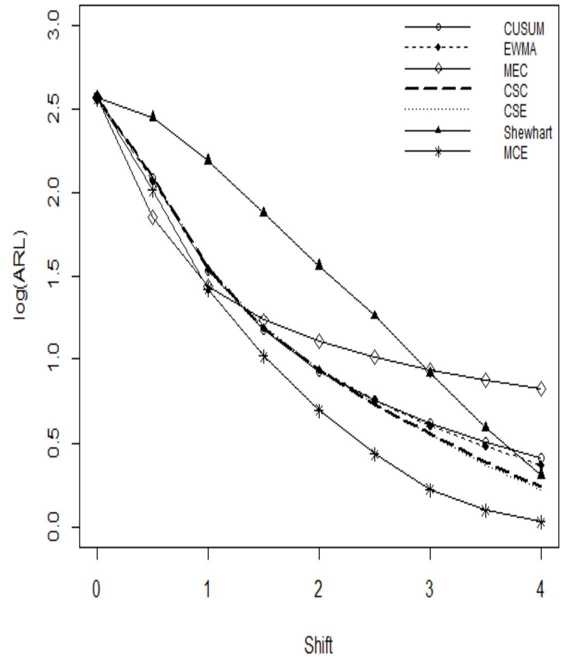
ϕ	δ	CSC	CSE	MEC	CUSUM	EWMA	Shewhart	MCE
0	0	372.21	374.95	374.32	370.95	371.47	370.43	369.12
	0.5	36.53	37.64	29.17	34.90	35.71	155.59	29.37
	1	10.13	10.14	13.92	9.96	9.86	43.64	7.77
	1.5	5.54	5.30	9.76	5.49	5.23	15.10	3.82
	2	3.77	3.53	7.81	3.84	3.57	6.40	2.35
	2.5	2.79	2.62	6.61	3.00	2.78	3.20	1.66
	3	2.15	2.03	5.79	2.48	2.31	2.00	1.32
	3.5	1.69	1.63	5.20	2.15	2.02	1.46	1.14
	4	1.35	1.33	4.74	1.96	1.81	1.20	1.06
	EQL	14.38	13.86	33.70	16.19	15.28	25.95	9.73
0.25	0	371.64	372.25	367.61	368.92	371.29	368.76	367.36
	0.5	63.69	65.16	41.84	62.02	62.66	212.95	51.40
	1	16.17	16.88	17.96	15.88	16.25	79.67	12.51
	1.5	8.00	7.91	12.10	7.86	7.72	31.46	5.46
	2	5.06	4.83	9.42	5.09	4.88	13.56	3.02
	2.5	3.54	3.33	7.81	3.76	3.51	6.41	1.92
	3	2.56	2.42	6.77	2.95	2.73	3.25	1.42
	3.5	1.94	1.82	6.01	2.44	2.26	1.86	1.16
	4	1.47	1.43	5.43	2.10	1.92	1.33	1.06
	EQL	18.86	18.32	40.02	20.78	19.81	45.09	12.14
0.5	0	370.64	370.03	371.50	370.57	373.04	373.14	369.30
	0.5	124.86	126.04	72.30	122.09	118.65	281.82	104.28
	1	35.74	36.55	27.92	34.39	35.26	155.52	26.92
	1.5	15.21	15.70	17.27	14.96	15.37	75.76	10.46
	2	8.58	8.62	12.74	8.51	8.69	36.62	4.96
	2.5	5.40	5.31	10.23	5.68	5.64	18.26	2.71
	3	3.60	3.50	8.67	4.15	4.00	8.23	1.67
	3.5	2.43	2.35	7.54	3.18	3.02	3.89	1.25
	4	1.74	1.67	6.70	2.55	2.34	2.03	1.07
	EQL	30.67	30.58	52.96	33.10	32.65	99.38	19.02
0.75	0	372.81	367.22	371.71	369.14	374.74	368.99	369.62
	0.5	254.53	256.65	176.23	248.91	246.11	340.48	233.68
	1	124.03	127.94	72.39	120.40	118.97	278.85	98.04
	1.5	60.05	61.16	39.14	58.70	58.76	202.28	40.40
	2	31.23	33.20	25.84	30.71	32.61	128.75	17.28
	2.5	16.99	18.45	18.95	18.35	19.52	79.20	7.47
	3	9.49	10.14	14.87	10.95	12.14	41.60	3.51
	3.5	5.35	5.69	12.26	7.21	7.72	19.32	1.86
	4	2.99	3.14	10.37	5.00	5.14	8.04	1.25
	EQL	91.10	95.49	99.16	97.39	101.25	313.06	53.44
0.9	0	367.95	372.95	368.22	368.32	370.32	368.61	374.28
	0.5	348.31	349.73	321.05	341.38	338.49	361.48	328.62
	1	282.32	284.35	208.82	284.59	281.10	346.13	244.32
	1.5	214.37	219.47	137.76	210.75	209.98	309.44	159.49
	2	151.76	151.34	91.01	153.31	156.03	259.59	89.19
	2.5	99.73	103.94	63.93	106.71	110.09	198.20	44.72
	3	59.61	60.54	46.76	75.20	76.47	129.35	19.09
	3.5	32.09	34.93	35.31	48.20	51.32	69.58	7.35
	4	15.90	16.76	26.92	31.33	32.20	31.84	2.75
	EQL	392.37	403.42	303.93	455.28	465.61	710.13	200.68

Table 2.2: *ARL* and *EQL* values for different residual charts at different shifts (δ) when $\phi < 0$

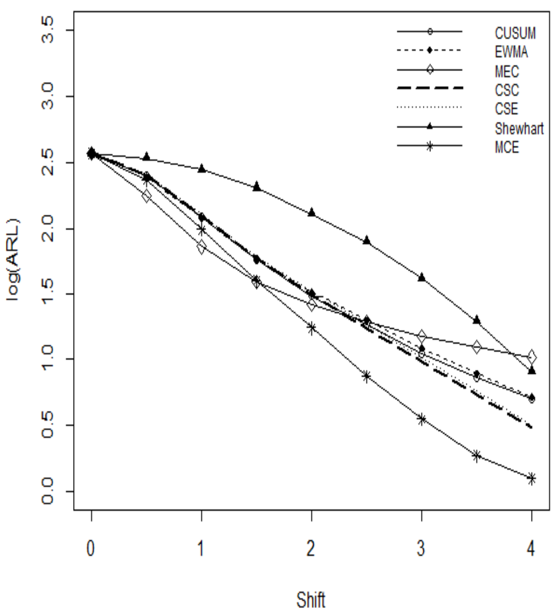
ϕ	δ	CSC	CSE	MEC	CUSUM	EWMA	Shewhart	MCE
-0.25	0	373.31	367.77	370.16	372.57	372.84	372.30	367.55
	0.5	23.23	24.80	22.61	22.86	23.68	113.18	19.59
	1	7.43	7.19	11.61	7.26	7.03	25.59	5.64
	1.5	4.39	4.10	8.43	4.41	4.10	8.20	3.04
	2	3.12	2.91	6.85	3.24	2.99	3.75	2.05
	2.5	2.40	2.27	5.86	2.62	2.43	2.26	1.55
	3	1.90	1.83	5.19	2.24	2.10	1.66	1.28
	3.5	1.55	1.53	4.67	2.03	1.93	1.33	1.14
	4	1.32	1.31	4.23	1.91	1.78	1.16	1.05
	EQL	12.15	11.77	29.76	14.07	13.26	17.75	8.65
-0.5	0	369.21	369.00	371.40	367.71	373.22	373.03	367.08
	0.5	16.81	17.62	18.59	16.42	16.79	81.54	13.98
	1	5.95	5.68	10.11	5.88	5.57	15.52	4.51
	1.5	3.70	3.43	7.52	3.74	3.48	5.11	2.65
	2	2.71	2.56	6.17	2.87	2.64	2.68	1.90
	2.5	2.13	2.06	5.33	2.38	2.22	1.89	1.50
	3	1.77	1.74	4.76	2.09	2.01	1.53	1.27
	3.5	1.51	1.51	4.24	1.98	1.91	1.31	1.14
	4	1.31	1.31	3.99	1.90	1.78	1.16	1.06
	EQL	10.94	10.70	27.06	12.88	12.21	13.63	8.10
-0.75	0	369.81	369.20	370.10	369.45	371.61	371.21	369.53
	0.5	13.04	13.25	16.24	12.74	12.70	59.70	10.72
	1	5.04	4.73	9.11	5.02	4.68	10.19	3.87
	1.5	3.21	3.03	6.87	3.34	3.08	3.60	2.38
	2	2.41	2.30	5.69	2.62	2.41	2.21	1.80
	2.5	1.97	1.92	4.95	2.21	2.10	1.76	1.48
	3	1.71	1.70	4.38	2.03	1.97	1.51	1.27
	3.5	1.51	1.50	4.02	1.96	1.90	1.31	1.13
	4	1.32	1.31	3.83	1.90	1.78	1.16	1.05
	EQL	10.24	10.03	25.19	12.19	11.60	11.50	7.76
-0.9	0	370.84	369.50	370.02	372.47	372.14	368.76	370.62
	0.5	11.30	11.53	15.00	11.16	11.17	50.01	9.43
	1	4.57	4.34	8.62	4.60	4.32	8.17	3.55
	1.5	3.00	2.84	6.54	3.16	2.91	3.11	2.29
	2	2.28	2.20	5.44	2.51	2.32	2.06	1.75
	2.5	1.91	1.89	4.76	2.15	2.05	1.73	1.47
	3	1.69	1.69	4.19	2.00	1.97	1.51	1.27
	3.5	1.50	1.50	3.96	1.96	1.90	1.31	1.14
	4	1.31	1.31	3.66	1.90	1.78	1.16	1.05
	EQL	9.92	9.78	24.25	11.90	11.37	10.70	7.63



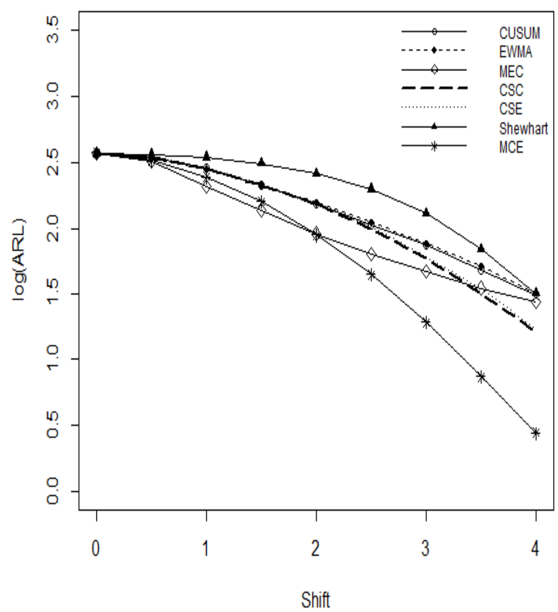
(a) $\phi = 0.25$



(b) $\phi = 0.50$

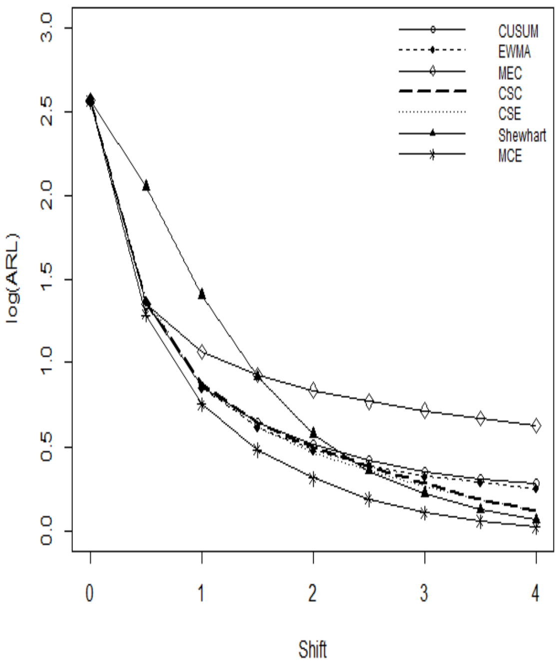


(c) $\phi = 0.75$

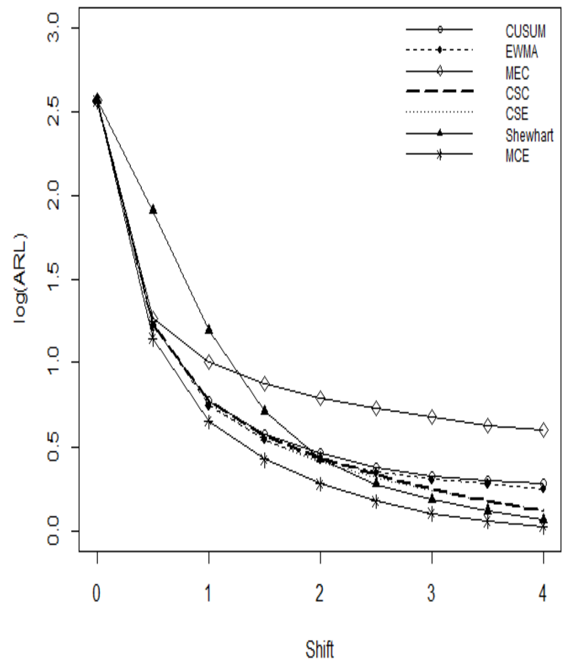


(d) $\phi = 0.90$

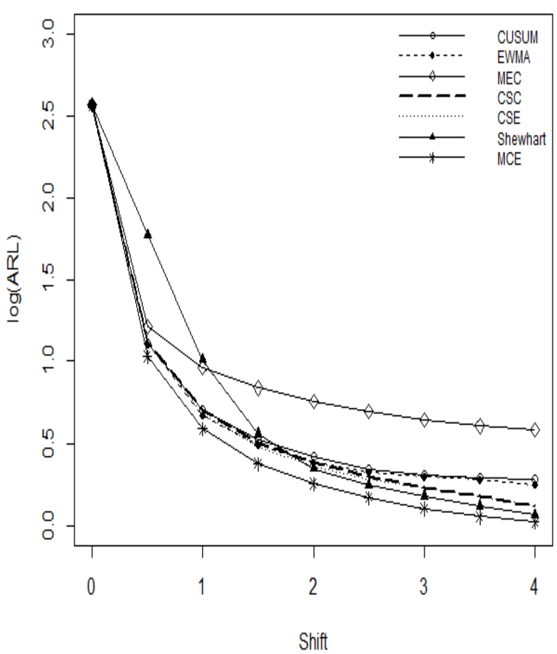
Figure 2.1: *ARL* curves for different residual charts for positive correlated processes



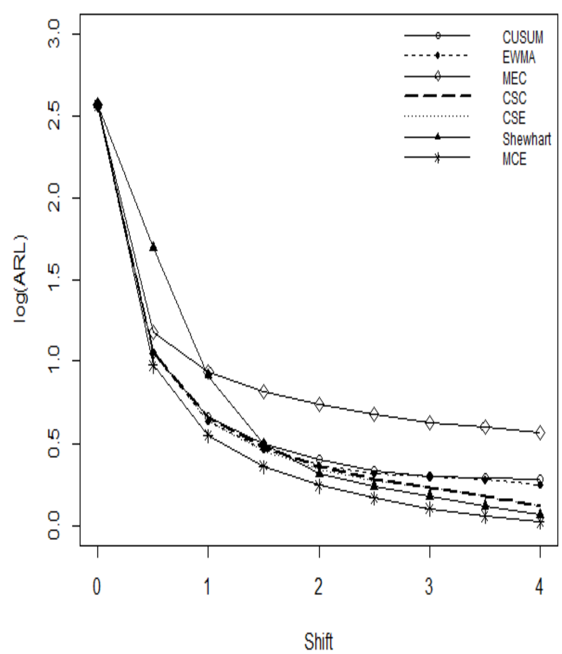
(a) $\phi = -0.25$



(b) $\phi = -0.50$



(c) $\phi = -0.75$



(d) $\phi = -0.90$

Figure 2.2: *ARL* curves for different residual charts for negative correlated processes

Table 2.3: *ARL* and *EQL* values for different residual charts at different shifts ($-\delta$) when $\phi = 0.25$

ϕ	- δ	CSC	CSE	MEC	CUSUM	EWMA	Shewhart	MCE
0.25	0	368.09	373.98	371.22	370.67	373.53	371.39	367.30
	0.5	63.82	65.59	41.64	61.17	62.30	213.42	52.84
	1	16.33	16.70	18.07	15.85	16.19	79.88	12.74
	1.5	7.91	7.88	12.12	7.82	7.63	32.10	5.61
	2	5.04	4.83	9.40	5.13	4.84	13.81	3.01
	2.5	3.55	3.33	7.82	3.76	3.52	6.38	1.94
	3	2.57	2.44	6.78	2.93	2.75	3.24	1.41
	3.5	1.91	1.82	6.01	2.45	2.27	1.88	1.16
	4	1.48	1.43	5.43	2.10	1.92	1.34	1.06
	EQL	18.86	18.32	40.04	20.74	19.77	45.43	12.28

2.4. Performance Comparison of Control Charts

In this section, all the residual charts are compared with each other using the *ARL* and *EQL* performance criteria. We group the comparison discussion under uncorrelated, positively and negatively correlated processes.

2.4.1. *ARL*

Uncorrelated observations

In Table 2.1, it can be observed that when observations from the process are uncorrelated ($\phi = 0$), the MCE residual chart performs better than all the other charts for all levels of shifts in the process mean. However, the MEC,CSE,CSC,EWMA and CUSUM residual charts all perform quite well for small shifts and the Shewhart, CSE and CSC also perform creditably well for large shifts in the process.

Positively correlated observations

It can be seen in Table 2.1 that for positively correlated processes ($\phi > 0$), the MEC residual chart followed by the MCE residual charts outperform the other charts for small shifts in the process mean. Take note that the CUSUM, EWMA, CSE and CSC residual charts perform the same and almost quite well for small shifts. The Shewhart residual chart is the worst performing chart for small shifts.

Again the MCE performs better than the other charts with respect to medium shifts ($1 \leq \delta < 3$). It is worth pointing out that when $\phi = 0.75$ and $\phi = 0.90$ the MEC residual chart also performs creditably well for medium shifts especially between $1 \leq \delta < 2$. The performance of the CUSUM, EWMA, CSE and CSC residual charts are almost the same for medium shifts but inferior to the MCE residual chart.

With large shifts in the mean of positive correlated process, the MCE performs exceptionally well and better than the other charts. Generally, the CSE and CSC residual charts also perform better than the EWMA, CUSUM and Shewhart residual charts. For highly correlated process (i.e. $\phi \geq 0.75$), the performance of the Shewhart residual chart is worst for large shifts.

Negatively correlated observations

In Table 2.2, it is noted that when a process is negatively autocorrelated ($\phi < 0$), all the charts (MCE, MEC, EWMA, CUSUM, CSE and CSC) except the Shewhart residual chart perform creditably well for small shifts in the process. However, the MEC residual chart mostly performs well for $\delta \leq 0.5$.

Furthermore, the MCE residual chart performs better than the other charts for medium shifts in the process mean. The performance of CSE, CSC, EWMA and CUSUM residual charts are almost the same for medium shifts but inferior to the MCE residual chart. The Shewhart residual chart does perform quite well like both the CSC and CSE residual charts with $2 < \delta < 3$. The MEC chart is the worst performing chart for this shift category.

For large shifts, the MCE and MEC residual chart are the best and worst residual charts for negatively correlated process respectively. However, the Shewhart, CSE and CSC residual charts perform equally well for large shifts as expected.

2.4.2. *EQL*

Lastly with in-control *ARL* fixed at 370 in all the competing charts, it is observed from Tables 2.1 and 2.2 that the MCE residual chart followed by the CSE and CSC residual charts consistently has the smallest *EQL* values for both positive and negative autocorrelated process. This result indicates that the MCE residual chart has the best performance ability over the shift range with respect to correlated processes. It is clear that for positive and negative correlated processes, the Shewhart and MEC residual charts respectively are the worst performing charts.

2.5. Illustrative Examples

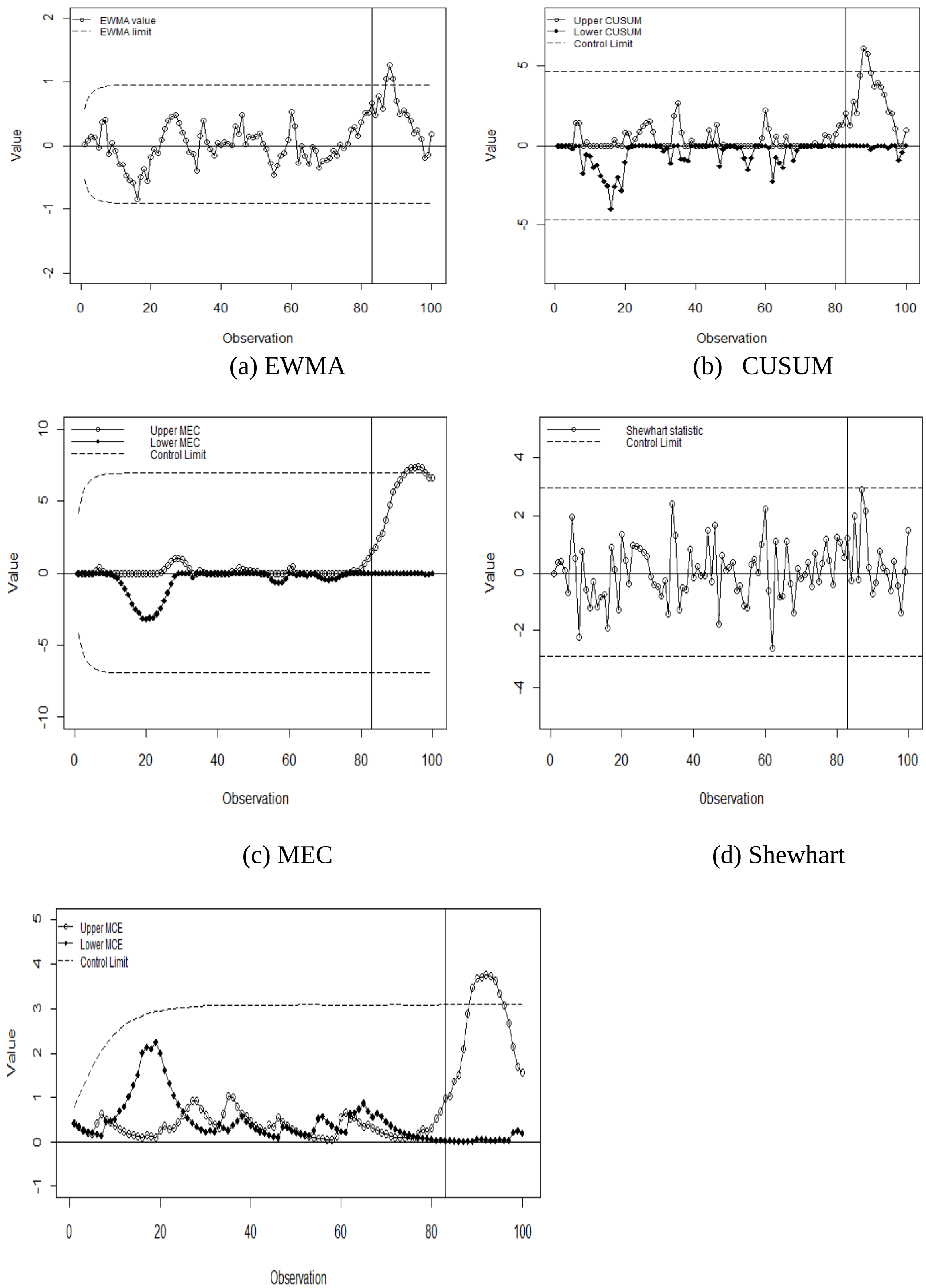
In this section, we considered both the simulated and real life data sets for the illustration of the control charts when a sustained shift is introduced in the process mean of the AR (1) observations.

(a) Example 1: Simulated data

We simulated 100 observations from an AR (1) process model with $\phi = 0.5$. The residuals of this model are standardized. At observation 83, a persistent shift of magnitude $1\sigma_e$ is introduced in the observations. The first 82 observations represent the in-control state of the process. Based on these residuals, we constructed different charts as described in section 2.2. The various parameters in the residual charts are chosen such that the in-control *ARL*= 370. The graphical displays of the charts are presented in Figures 2.3(a-e). We have omitted the graphical displays for the CSE and CSC residual charts because their outputs are similar to the individual EWMA and CUSUM residual charts respectively. Table 2.4 displays the various charts with their corresponding out-of-control points. Due to lack of space we have provided the first 35 out of the 100 values of each charting statistic computed in Table 2.5.

Table 2.4: Out-of-control points for the residual charts based on data in Example 1

Charts	Out of control points
Shewhart	0
EWMA	87, 88 ,89
CUSUM	88 ,89
MEC	93 ,94 ,95, 96 ,97 ,98
MCE	89 ,90 ,91, 92,93,94 ,95



(a) EWMA (b) CUSUM

(c) MEC (d) Shewhart

(e) MCE

Figure 2.3: Residual charts for simulated data in Example 1

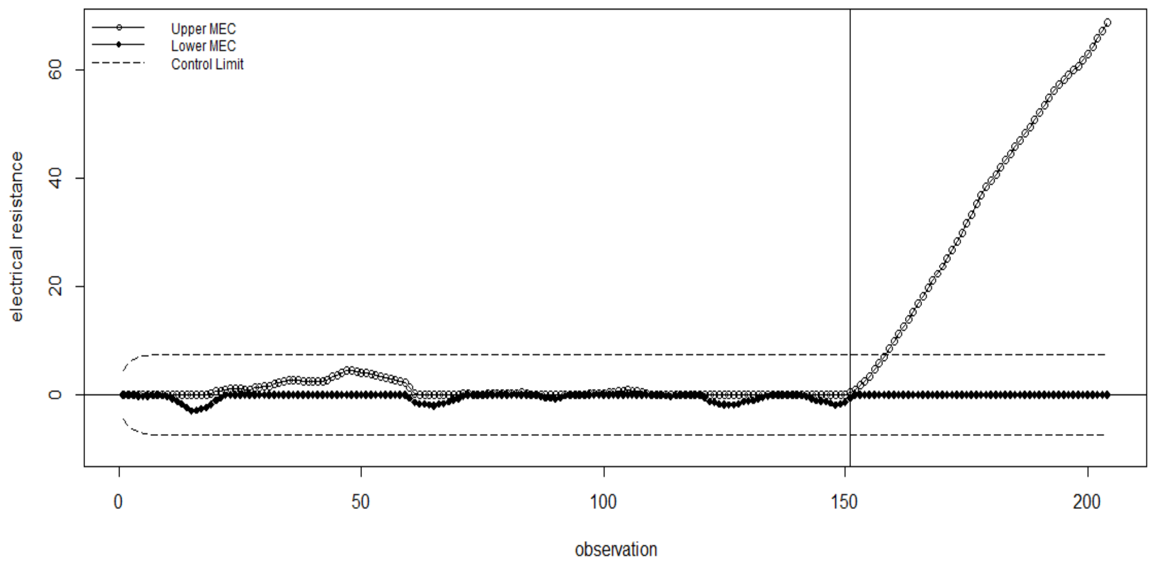
Table 2.5: The values of the charting statistics (35 out of 100 observations)

Observation	Shewhart	EWMA	CUSUM		MEC		MCE	
	e_t	W_t	C_t^+	C_t^-	MEC_t^+	MEC_t^-	MCE_t^+	MCE_t^-
1	0.00	0.02	0.00	0.00	0.00	0.00	0.40	0.40
2	0.36	0.09	0.00	0.00	0.00	0.00	0.32	0.32
3	0.41	0.15	0.00	0.00	0.00	0.00	0.26	0.26
4	0.09	0.14	0.00	0.00	0.00	0.00	0.21	0.21
5	-0.66	-0.02	0.00	0.19	0.00	0.00	0.17	0.20
6	1.96	0.38	1.45	0.00	0.19	0.00	0.42	0.16
7	0.51	0.40	1.45	0.00	0.42	0.00	0.63	0.13
8	-2.24	-0.12	0.00	1.77	0.11	0.00	0.50	0.46
9	0.75	0.05	0.24	0.55	0.00	0.00	0.45	0.48
10	-0.56	-0.07	0.00	0.64	0.00	0.00	0.36	0.51
11	-1.22	-0.30	0.00	1.40	0.00	0.16	0.29	0.69
12	-0.27	-0.30	0.00	1.21	0.00	0.32	0.23	0.79
13	-1.18	-0.47	0.00	1.91	0.00	0.65	0.18	1.02
14	-0.84	-0.55	0.00	2.29	0.00	1.06	0.15	1.27
15	-0.73	-0.58	0.00	2.55	0.00	1.50	0.12	1.53
16	-1.91	-0.85	0.00	3.99	0.00	2.20	0.09	2.02
17	0.91	-0.50	0.40	2.61	0.00	2.56	0.16	2.14
18	0.14	-0.37	0.02	2.01	0.00	2.79	0.13	2.11
19	-1.30	-0.56	0.00	2.84	0.00	3.21	0.10	2.26
20	1.37	-0.17	0.86	1.01	0.00	3.24	0.25	2.01
21	0.44	-0.05	0.78	0.10	0.00	3.14	0.36	1.63
22	-0.37	-0.11	0.00	0.00	0.00	3.12	0.29	1.30
23	0.96	0.10	0.45	0.00	0.00	2.87	0.32	1.04
24	0.94	0.27	0.88	0.00	0.08	2.46	0.43	0.83
25	0.88	0.39	1.24	0.00	0.29	1.93	0.59	0.67
26	0.73	0.46	1.46	0.00	0.56	1.33	0.77	0.53
27	0.58	0.48	1.52	0.00	0.86	0.71	0.92	0.43
28	-0.11	0.36	0.90	0.00	1.03	0.21	0.91	0.34
29	-0.38	0.22	0.00	0.00	1.06	0.00	0.73	0.27
30	-0.46	0.08	0.00	0.00	0.96	0.00	0.58	0.22
31	-0.81	-0.10	0.00	0.34	0.67	0.00	0.47	0.24
32	-0.26	-0.13	0.00	0.14	0.36	0.00	0.37	0.22
33	-1.44	-0.39	0.00	1.11	0.00	0.25	0.30	0.40
34	2.39	0.16	1.88	0.00	0.00	0.00	0.62	0.32
35	1.31	0.39	2.67	0.00	0.21	0.00	1.03	0.26

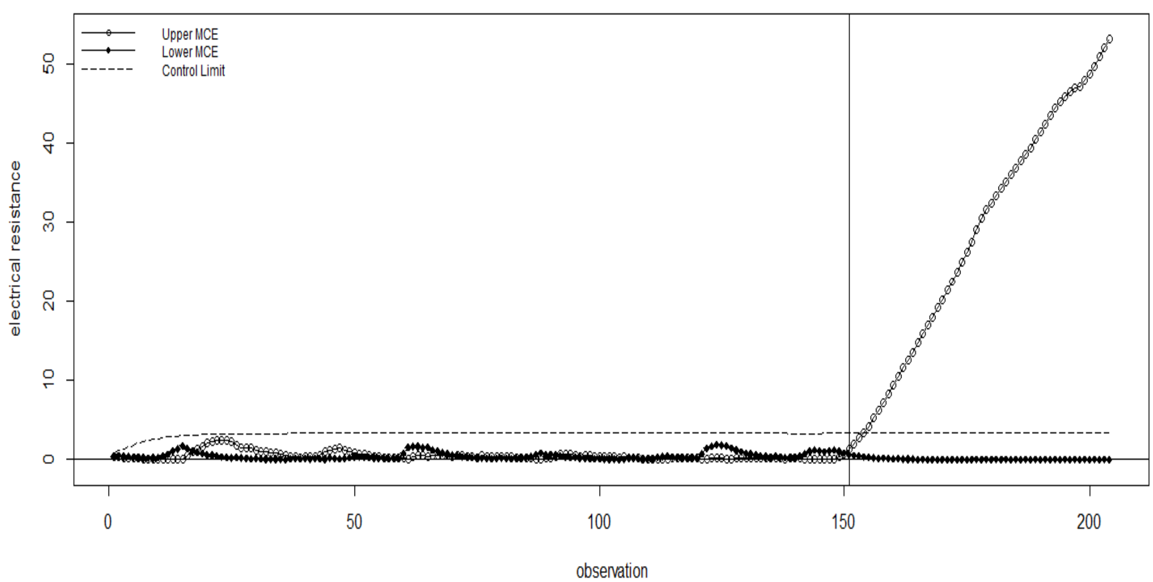
It is clear from Table 2.4 that after the 82nd observation, the MCE, MEC, EWMA, CUSUM and Shewhart residual charts detected 7,6,3,2 and 0 out-of-control points respectively. The proposed charts were slow and did not detect the initial two out-of-control signals as compared to the CUSUM and EWMA residual charts but in all the proposed charts detected the most signals. From this example the MCE residual chart was the best performing chart, followed by the MEC and the EWMA residual charts. The observations in this example are consistent with the findings in Section 2.4.

(b) Example 2: Real life data

In this example, we use a real life dataset to demonstrate how the newly proposed charts perform. The dataset from Shewhart (1931) consist of 204 electrical resistance measurements. An AR (1) process model with $\phi = 0.55$ provided a perfect fit to the data. The residuals obtained from the model are standardized. At observation 151, a step shift of $2.5\sigma_e$ is introduced into the observations. Figure 2.4(a-b) displays the outputs from the MEC and MCE residual charts respectively. After the 150th observation, the MCE and MEC charts signalled 51 and 46 out-of- control points respectively. This shows the superiority of the MCE residual chart over the MEC residual chart with respect to medium shifts. The findings in this example are also consistent with the results in Section 2.4.



(a) MEC



(b) MCE

Figure 2.4: Residual charts for real life data in Example 2

2.6. Conclusions

Many situations arise in manufacturing and industrial operations when the observations become correlated. In this chapter, we investigated and compared the performances of a wide range of existing (Shewhart, CUSUM, EWMA, CSC and CSE) and newly proposed (MEC and MCE) residual charts for monitoring autocorrelated data. The advantage of using the MEC residual chart is that it performs better than the existing charts with respect to small mean shifts for positively correlated processes. The MCE residual chart on the other hand performs better than the existing charts for autocorrelated processes with step shift in the mean of the AR (1) process model. The MCE residual chart is recommended for monitoring autocorrelated processes that can be fitted with an AR (1) process model.

Future research can concentrate on the optimization designs of the MEC and MCE charts for correlated processes and also the application of theses charts on serially correlated multivariate data can be studied.

CHAPTER 3

MODIFIED CONTROL CHARTS FOR MONITORING AUTOCORRELATED DATA

3.1. Introduction

Statistical process control (SPC) tools have become relevant in production operations to monitor variation in the process parameters. Control charts are considered as the most prominent SPC tool. They are utilised to monitor a process based on the fundamental assumptions of independence and normality of the observations (Montgomery (2009)). In reality, this assumption of independence is not usually fulfilled because most production processes produce observations which are autocorrelated. Most chemical production processes produce highly correlated observations due to the depletion of vital catalyst, contamination of tools, etc. (cf. Mason and Young (2002)). With the occurrence of autocorrelation the control charts do not function as anticipated and produce frequent false alarms or respond slowly to out of control states (Alwan and Roberts (1988) , Harris and Ross (1991), Montgomery and Mastrangelo (1991), Zhang (2000), Montgomery (2009)).

The autocorrelation is accounted for by using two of the widely accepted methods. These methods involve the use of the residuals and modified charts. In the residual charts, the control charts are applied on the residuals obtained from fitting a time series model to the correlated data. Works by Alwan and Roberts (1988), Harris and Ross

(1991), Montgomery and Mastrangelo(1991), English *et al.* (2000), Karoglan and Bayhan (2011), Areepong (2013), etc. all concentrated on the use of the residual charts to handle the issue of autocorrelation. However, in modified charts the control limits of the traditional charts are adjusted to account for the autocorrelation in the observations. Vasilopoulos and Stamboulis (1978) advanced the idea of modifying the standard control limits of the Shewhart chart when the observations from the process are correlated. Schmid (1995) extended the work of Vasilopoulos and Stamboulis (1978) by probing into the run length of the Shewhart chart as applied to the correlated data. He analytically proved that the in-control average run length for the correlated data is larger than the case when observations are independent. Wieringa (1999), Vermat *et al*. (2008) and VanBrackle and Reynolds (1997) derived variance expressions for the EWMA statistic for correlated data that can be fitted with an AR (1), AR (2) and ARMA (1, 1) process models respectively. With these evaluated variances, the control limits of the EWMA chart can be adjusted accordingly for the respective proposed models, in order to yield the desired in-control *ARL* value.

Other works also compared the performance of control charts based on the residuals and original observations in terms of their sensitivity and ability to detect shifts in the mean of autocorrelated processes. Such comparisons have been made by numerous authors, including Schmid (1995), Kramer and Schmid (1997), Wieringa (1999) and the references therein. Wardel *et al*. (1992) handled the subject by comparing the performances of the Shewhart, EWMA, Special cause and Common cause charts using ARMA (1, 1) process model. In the study, they recognized that the control charts are not completely robust to deviations from the assumptions cited for the use of the charts.

Additionally, they remarked that the conventional EWMA chart is very responsive to small means shifts and does perform creditably well for large shifts when the AR and MA parameters are both negative whilst the Shewhart type charts proposed by Alwan and Roberts (1988) performed suitably well in most cases for large shifts in the process. In a performance comparison between the modified and residual Shewhart charts, Kramer and Schmid (1997) demonstrated that the modified Shewhart chart was preferred when the process was positively correlated and the residual Shewhart chart performed better for negative correlated processes. Zhang (1998) devised the EWMAST chart which is basically a EWMA chart for stationary processes. Subsequently, Zhang (2000) extended his earlier work to include a performance comparison between the EWMAST, Shewhart chart of observations, Shewhart, CUSUM and EWMA charts of residuals. Overall the EWMAST chart showed superiority in detecting small to medium shifts for autocorrelated processes.

Lu and Reynolds (1999) considered the application of the EWMA control charts on autocorrelated observations. The work centred on monitoring the mean of observations that can be represented as an AR (1) process model plus a random error term based on the forecast errors of the model and the original observations. Lu and Reynolds (2001) conducted a similar work using the CUSUM control charts. They observed that the CUSUM chart of observations and residuals perform equally well for small shifts when the process is highly correlated but the CUSUM chart of residuals performed relatively better than the CUSUM chart of observations for large mean shifts. Knoth and Schmid (2004) also compared the performance of the EWMA and CUSUM schemes using original observations and residuals from an AR (1) model in a review work with respect to shifts in the mean of the correlated data. Lin (1995) proposed the combined Shewart-CUSUM (CSC) and combined Shewhart-EWMA (CSE) charts for autocorrelated data using the residuals of the underlying time series model. Using correlated observations, their respective combined modified charts can be used in process monitoring. It is noted that the performance of control charts for correlated data varies and depends on many parameters such as the level of the autocorrelation, the type of chart, the process model used, etc.

As discussed in Section 2.2.2, the MEC chart is best noted for monitoring small shifts in the process whilst the MCE chart performs exceptionally well for detecting small,medium and large shifts in the process mean. The work on these charts for autocorrelated observations have not been done so far in SPC literature. In this chapter, we propose the MEC and MCE modified charts to correct the problems associated with serial correlation. The aim of this chapter is to investigate how the newly proposed modified charts respond to serial correlation as compared to the existing (Shewhart, CUSUM, EWMA, combined Shewhart-CUSUM (CSC) and combined Shewhart-EWMA (CSE)) charts with shifts in the process mean.

The rest of the chapter is organised as follows: Section 3.2 deals with the modelling of serially correlated processes and discusses the existing and proposed modified charts; the performance evaluation procedures are briefly examined in Section 3.3; comparison of the charts are summarised in Section 3.4. Section 3.5 presents an illustrative example and lastly conclusions are presented in Section 3.6.

3.2. Modelling Serially Correlated Processes

Serial correlation has negative effects on the performance of control charts. An effect of ignoring this behaviour in process monitoring leads to regular false alarms when the process is actually stable. We consider autocorrelation that can be fitted with an AR (1) process model. This model is discussed in Section 2.2 and it is expressed as;

$$X_t - \mu = \phi_1 (X_{t-1} - \mu) + \varepsilon_t \tag{3.1}$$

To model assignable causes in the charts, a shift of $\delta\sigma_X$ is introduced into the process model such that μ shifts to $\mu + \delta\sigma_X$. Without restriction, if we assume $\mu = 0$ and $\sigma_\varepsilon = 1$ then we obtain;

$$\left.\begin{aligned} W_t &= X_t + \delta\sigma_X \\ &= \phi_1 X_{t-1} + \varepsilon_t + \delta\sigma_X \end{aligned}\right\} \tag{3.2}$$

Therefore W_t becomes the current observation which is used in the charts instead of X_t.

The variance is given as;

$$\text{var}(W_t) = \sigma_W^2 = \sigma_X^2 = \frac{\sigma_\varepsilon^2}{(1-\phi_1^2)} \tag{3.3}$$

We briefly describe the structures of the charts under consideration in the next subsection.

3.2.1 Existing Control Charts

In this section, we briefly discuss the control chart schemes for the existing modified charts for dealing with correlated observations. The Shewhart, CUSUM, EWMA, CSC and CSE modified charts are discussed.

Shewhart modified chart

With the existence of serial correlation the Shewhart chart is modified to account for the correlation by adjusting the standard control limits of the chart (see Vasilopoulos and Stamboulis (1978), Schmid (1995), Kramer and Schmid (1997), Wieringa (1999)). In the implementation of the modified chart, W_t in equation (3.2) becomes the current observation. The chart is considered out-of-control when W_t falls outside the upper control and lower control limits.

$$UCL = \mu + L_S \sigma_W \quad \text{and} \quad LCL = \mu - L_S \sigma_W \tag{3.4}$$

Where L_S is a control chart multiplier which is adjusted to achieve the desired *ARL* value.

CUSUM modified chart

The CUSUM chart is transformed into the CUSUM modified chart by modifying the standard control limit when the observations are correlated. In the CUSUM modified chart (see Wieringa (1999)), the 2 sided CUSUM statistics are expressed as;

$$\left.\begin{aligned} C_t^+ &= \max(0, W_t - u - K + C_{t-1}^+) \\ C_t^- &= \max(0, u - W_t - K + C_{t-1}^-) \end{aligned}\right\} \tag{3.5}$$

Where $K = k\sigma_W$ and $H = h\sigma_W$. This modified chart is out-of-control when either of the CUSUM statistics exceed H, i.e.

$$C_t^+ > H \quad \text{or} \quad C_t^- > H \tag{3.6}$$

EWMA modified chart

When observations are correlated, the EWMA modified chart is used to handle the effects due to the autocorrelation in process monitoring. In this modified chart (see Wieringa (1999)), the standard control limits of EWMA chart are adjusted. In this modified chart, the EWMA statistic (Z_t) is defined as:

$$Z_t = \lambda W_t + (1-\lambda)\,Z_{t-1} \tag{3.7}$$

The asymptotic variance (Wieringa (1999)) of the EWMA statistic for the AR (1) process is given as;

$$\operatorname{var}(Z_t) = \sigma_Z^2 \approx \frac{\sigma_\varepsilon^2}{(1-\phi_1^2)}\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi_1(1-\lambda)}{1-\phi_1(1-\lambda)}\right) \tag{3.8}$$

For the proof of the exact variance we refer the reader to Wieringa (1999)). We use the asymptotic variance of the EWMA statistic in this chapter. This chart is said to be out-of-control when Y_t falls outside the control limits, i.e.

$$UCL = \mu + L_E\sigma_W\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi_1(1-\lambda)}{1-\phi_1(1-\lambda)}\right)} \quad \text{and} \quad LCL = \mu - L_E\sigma_W\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi_1(1-\lambda)}{1-\phi_1(1-\lambda)}\right)} \tag{3.9}$$

Where L_E is the control constant which is adjusted to achieve the desired in-control run length value.

Combined Shewhart-CUSUM (CSC) modified chart

In the CSC modified chart, the Shewhart and CUSUM control limits are modified to account for the incidence of serial correlation. A process is identified to be out-of-control when W_t falls outside the Shewhart limits or any of the CUSUM statistics exceed H.

$$\left.\begin{array}{l} W_t > \mu + L_S\sigma_W \quad \text{or} \quad W_t < \mu - L_S\sigma_W \quad \text{or} \\ C_t^+ > H \quad \text{or} \quad C_t^- > H \end{array}\right\} \tag{3.10}$$

Where L_S is a control chart constant in the Shewhart chart.

Combined Shewhart-EWMA (CSE) modified chart

The CSE modified chart is used for location monitoring when observations are serially correlated. The CSE modified chart has the standard control limits of the Shewhart and EWMA schemes adjusted to account for the correlation. In this chart an out-of-control signal is obtained when Z_t falls outside the EWMA limits or W_t falls outside the Shewhart limits. That is;

$$\left.\begin{array}{l} W_t > \mu + L_S\sigma_W \quad \text{or} \quad W_t < \mu - L_S\sigma_W \quad \text{or} \\ Z_t > \mu + L_E\sigma_W\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi_1(1-\lambda)}{1-\phi_1(1-\lambda)}\right)} \quad \text{or} \quad Z_t < \mu - L_E\sigma_W\sqrt{\left(\frac{\lambda}{2-\lambda}\right)\left(\frac{1+\phi_1(1-\lambda)}{1-\phi_1(1-\lambda)}\right)} \end{array}\right\} \tag{3.11}$$

Where L_S and L_E are the control constants in the individual Shewhart and EWMA charts respectively.

3.2.2 Proposed Modified Control Charts

In this section, we discuss the control chart structures for autocorrelated processes using the two newly proposed charts (MEC and MCE modified charts).

Mixed EWMA-CUSUM (MEC) modified chart

The MEC chart also faces the usual problem of frequent false alarms associated with correlated data. Therefore, we propose the MEC modified chart for process monitoring of correlated data. The structure of the chart is represented as;

$$\left.\begin{aligned} MEC_t^+ &= \max(0, Z_t - u - K + MEC_{t-1}^+) \\ MEC_t^- &= \max(0, u - Z_t - K + MEC_{t-1}^-) \end{aligned}\right\} \tag{3.12}$$

Where $K = k\sigma_Z$ and $H = h\sigma_Z$. The modified chart reports an out-of-control condition when either of the MEC statistics exceed the decision interval value (H).

$$MEC_t^+ > H \quad \text{or} \quad MEC_t^- > H \tag{3.13}$$

Mixed CUSUM-EWMA (MCE) modified chart

The MCE chart has superior performance ability in detecting small, medium and large shifts in the mean of the process better than the existing individual Shewhart, EWMA and CUSUM charts. This chart is transformed into the MCE modified chart to improve process performance with the existence of serial correlation. The proposed chart is defined by these statistics;

$$\left.\begin{aligned} MCE_t^+ &= \lambda C_t^+ + (1-\lambda) MCE_{t-1}^+ \\ MCE_t^- &= \lambda C_t^- + (1-\lambda) MCE_{t-1}^- \end{aligned}\right\} \quad (3.14)$$

The initial values of the MCE statistics in equation (3.14) are fixed equal to the target mean of C_t^+ and C_t^- respectively for the correlated observations following Zaman *et al*. (2014), i.e. $MCE_0^+ = MCE_0^- = \mu_C$.

The mean and variance of the MCE statistics for the correlated observations are time varying and are obtained through a simulated process. These are represented as;

$$\left.\begin{aligned} \text{mean}(MCE_t^+) = \text{mean}(MCE_t^-) = \mu_{MCE_t} \\ \text{Var}(MCE_t^+) = \text{Var}(MCE_t^-) = \sigma^2_{MCE_t} \end{aligned}\right\} \quad (3.15)$$

The maximum value of *t* considered is 10,000 and the simulation process is repeated 10000 times in this paper to obtain the mean and variance of the MCE statistics. Take note that as $t \to \infty$, the target mean of the CUSUM statistics for the correlated observations become equal to the target mean of the MCE statistics, i.e. $\mu_{MCE} = \mu_C$. We say the process is out-of-control if either MCE_t^+ or MCE_t^- falls outside the control limit.

$$MCE_t^+ > UCL_t = \mu_{MCE_t} + L_M \sigma_{MCE_t} \quad \text{or} \quad MCE_t^- > UCL_t = \mu_{MCE_t} + L_M \sigma_{MCE_t} \quad (3.16)$$

Where L_M is the control constant in the chart.

3.3. Performance Evaluation

The evaluation of the modified charts is carried out based on the *ARL* and *EQL* measures. The *ARL* and *EQL* measures are two of the widely used measures in the assessment and comparison of control charts. These performance criteria are discussed in Section 1.3.

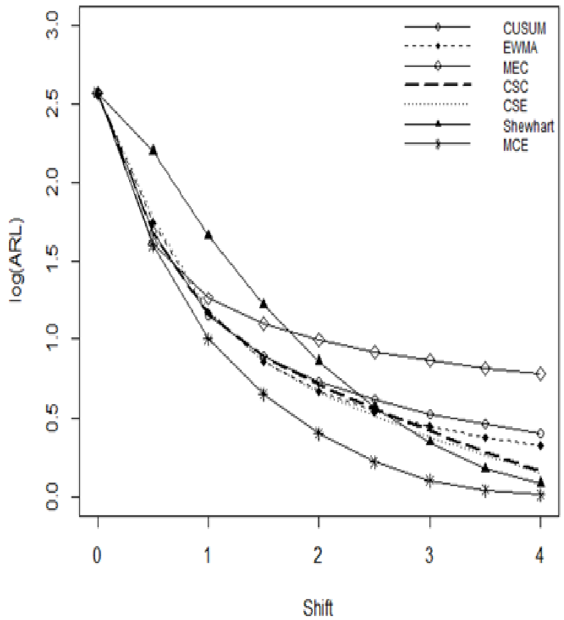
These performance measures are evaluated by considering shifts $\delta = 0, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4$ and correlation parameters $\phi = \pm 0.25, \pm 0.5, \pm 0.75, \pm 0.9$. The *ARL* values are generated by performing 10,000 simulations at each level of shift and autocorrelation value whilst the *EQL* values are estimated numerically using the Trapezium rule. For easy comparison we group the shifts into small ($\delta < 1$), moderate ($1 \le \delta < 3$) and large ($\delta \ge 3$) categories. All the charts are calibrated to yield $ARL_0 = 370$. To ensure the validity of the simulation processes, we have reproduced the *ARL* figures of the EWMAST and Shewhart charts of observations in Zhang (2000). These figures are consistent with the *ARL* values for the EWMA and Shewhart modified charts as referred to in this work. Tables 3.1 and 3.2 display the simulation results and the parameters used in the various modified charts. Figures 3.1 and 3.2 present the graphical displays for the *ARL* curves of the charts.

Table 3.1: *ARL* for the various modified control charts applied to AR (1) process with $\phi \geq 0$

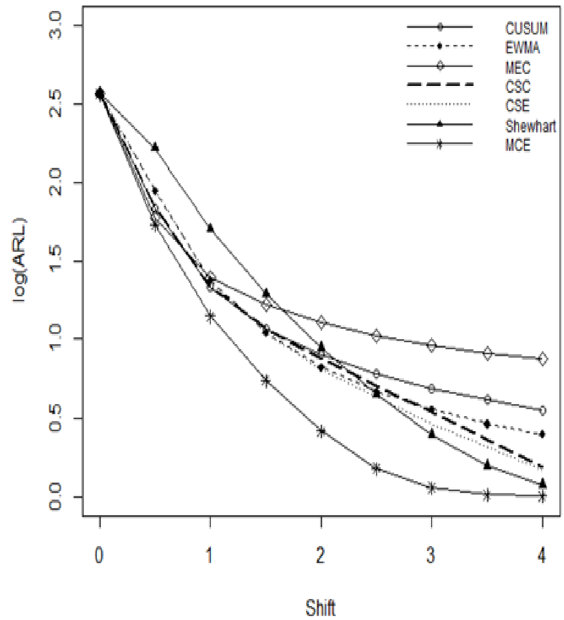
		CSC	CSE	MEC	CUSUM	EWMA	Shewhart	MCE
ϕ	δ	*L_S=3.5,k=0.5*	*L_S=3.5,λ=0.2*	*k=0.5, λ=0.2*	*k=0.5*	*λ=0.2*		*k=0.5,λ=0.2*
		h=4.914	*L_E=2.91*	*h=21.28*	*h=4.77*	*L_E=2.86*	*L_S=3*	*L_M=4.18*
0	0	372.21	374.95	374.32	370.95	371.47	370.43	369.12
	0.5	36.53	37.64	29.17	34.90	35.71	155.59	29.37
	1	10.13	10.14	13.92	9.96	9.86	43.64	7.77
	1.5	5.54	5.30	9.76	5.49	5.23	15.10	3.82
	2	3.77	3.53	7.81	3.84	3.57	6.40	2.35
	2.5	2.79	2.62	6.61	3.00	2.78	3.20	1.66
	3	2.15	2.03	5.79	2.48	2.31	2.00	1.32
	3.5	1.69	1.63	5.20	2.15	2.02	1.46	1.14
	4	1.35	1.33	4.74	1.96	1.81	1.20	1.06
	EQL	14.38	13.86	33.70	16.19	15.28	25.95	9.73
		h=7.02	*L_E=2.847*	*h=23.2*	*h =6.83*	*L_E=2.795*	*L_S=2.998*	*L_M=3.8*
0.25	0	370.40	371.67	374.85	374.97	370.08	371.26	373.61
	0.5	48.75	60.03	41.41	48.82	56.48	159.53	40.16
	1	14.44	15.25	18.27	14.23	14.75	46.03	10.06
	1.5	7.80	7.26	12.47	7.78	7.22	16.51	4.50
	2	5.14	4.59	9.88	5.36	4.66	7.22	2.53
	2.5	3.62	3.25	8.32	4.10	3.46	3.71	1.66
	3	2.64	2.39	7.30	3.36	2.79	2.21	1.27
	3.5	1.91	1.82	6.57	2.87	2.37	1.50	1.10
	4	1.46	1.42	6.00	2.50	2.11	1.21	1.03
	EQL	18.27	17.56	42.79	22.06	19.55	27.88	10.49
0.5		*h=10.626*	*L_E=2.74*	*h=25.8*	*h=10.31*	*L_E=2.715*	*L_S=2.975*	*L_M=3.35*
	0	374.91	371.31	371.63	374.18	369.10	369.63	368.18
	0.5	69.82	90.29	60.66	69.37	89.05	166.29	54.88
	1	21.77	24.36	25.08	22.03	24.24	50.98	14.02
	1.5	11.69	10.90	16.50	11.70	10.97	19.69	5.48
	2	7.54	6.46	12.71	7.99	6.61	8.79	2.61
	2.5	5.07	4.27	10.56	5.98	4.63	4.46	1.51
	3	3.45	2.92	9.18	4.86	3.57	2.48	1.14
	3.5	2.28	2.05	8.18	4.10	2.90	1.57	1.03
	4	1.54	1.47	7.44	3.55	2.49	1.19	1.01
	EQL	24.84	23.40	54.58	32.18	26.78	31.37	11.36
0.75		*h=18.6*	*L_E=2.575*	*h=31.3*	*h=18.18*	*L_E=2.57*	*L_S=2.899*	*L_M=2.75*
	0	374.95	372.64	371.73	373.51	373.12	373.05	372.57
	0.5	112.23	145.74	104.74	109.50	145.64	187.47	76.41
	1	38.92	46.55	41.91	38.88	47.37	67.50	17.30
	1.5	20.73	20.96	25.63	21.15	20.54	28.18	4.87
	2	13.02	11.00	18.74	14.06	11.27	12.69	1.73
	2.5	8.46	6.55	15.22	10.31	7.13	6.10	1.10
	3	5.23	4.04	12.95	8.23	5.02	2.85	1.01
	3.5	2.89	2.45	11.34	6.85	3.86	1.51	1.00
	4	1.63	1.51	10.19	5.88	3.19	1.10	1.00
	EQL	39.27	36.69	79.11	54.94	42.21	39.94	11.31
0.9		*h=32.74*	*L_E=2.367*	*h=41.77*	*h=32.7*	*L_E=2.361*	*L_S=2.705*	*L_M=2.08*
	0	373.16	371.12	372.33	375.81	373.48	374.93	368.18
	0.5	174.86	207.88	170.99	173.61	206.49	220.70	97.29
	1	72.59	85.04	75.31	72.94	83.06	94.47	12.86
	1.5	38.97	39.12	43.46	39.78	38.98	41.55	1.81
	2	24.11	19.79	30.29	25.72	19.68	19.04	1.01
	2.5	15.78	10.71	23.38	18.59	10.85	7.42	1.00
	3	9.13	5.89	19.34	14.57	6.69	2.37	1.00
	3.5	3.90	2.82	16.66	11.99	4.69	1.13	1.00
	4	1.52	1.34	14.72	10.21	3.70	1.00	1.00
	EQL	67.65	58.69	122.38	98.08	64.52	51.11	10.10

Table 3.2: *ARL* for the various modified control charts applied to AR (1) process with $\phi < 0$

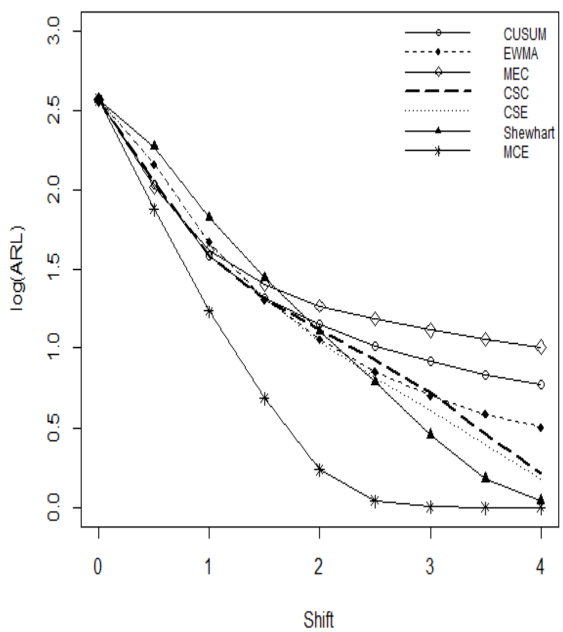
		CSC	CSE	MEC	CUSUM	EWMA	Shewhart	MCE
ϕ	δ	L_S=3.5,k=0.5	L_S=3.5,λ=0.2	k=0.5, λ=0.2	k=0.5	λ=0.2		k=0.5,λ=0.2
-0.25		h=3.52	L_E=2.955	h=19.43	h=3.44	L_E=2.91	L_S=2.998	L_M=4.42
	0	373.92	370.49	371.10	370.73	369.77	372.17	368.06
	0.5	27.22	24.14	21.44	26.19	22.88	155.83	21.31
	1	7.23	6.96	11.03	7.08	6.83	43.73	5.72
	1.5	4.08	3.98	8.07	4.03	3.97	14.29	3.11
	2	2.87	2.83	6.57	2.88	2.85	5.92	2.06
	2.5	2.21	2.20	5.66	2.29	2.29	3.02	1.56
	3	1.80	1.79	5.04	1.93	1.94	1.85	1.29
	3.5	1.51	1.51	4.58	1.70	1.70	1.38	1.13
	4	1.31	1.31	4.16	1.48	1.48	1.16	1.05
	EQL	11.72	11.52	28.87	12.31	12.16	25.03	8.74
-0.5		h=2.66	L_E=2.988	h=17.1	h=2.66	L_E=2.944	L_S=2.979	L_M=4.34
	0	371.28	372.88	370.88	371.96	373.33	371.03	368.56
	0.5	21.60	14.59	15.50	21.70	14.15	151.96	14.96
	1	5.49	4.98	8.56	5.44	4.86	42.98	4.13
	1.5	3.18	3.09	6.37	3.18	3.05	14.75	2.31
	2	2.29	2.29	5.27	2.30	2.27	6.00	1.65
	2.5	1.85	1.90	4.56	1.87	1.86	2.86	1.33
	3	1.59	1.63	4.08	1.58	1.60	1.73	1.14
	3.5	1.35	1.39	3.83	1.35	1.37	1.30	1.05
	4	1.17	1.19	3.38	1.16	1.18	1.12	1.01
	EQL	9.88	9.72	23.37	9.87	9.57	24.57	7.41
-0.75		h=2.41	L_E=2.972	h=12.87	h=2.41	L_E=2.955	L_S=2.90	L_M=3.975
	0	371.59	370.44	372.23	374.46	370.31	369.54	369.14
	0.5	27.44	8.45	10.02	27.51	8.31	145.35	12.14
	1	4.98	3.47	5.96	4.96	3.44	45.78	2.69
	1.5	2.92	2.29	4.54	2.93	2.28	16.65	1.54
	2	2.09	1.80	3.86	2.11	1.79	6.62	1.20
	2.5	1.74	1.49	3.21	1.74	1.47	2.86	1.06
	3	1.44	1.22	3.00	1.45	1.21	1.56	1.01
	3.5	1.19	1.06	2.97	1.19	1.06	1.19	1.00
	4	1.05	1.01	2.64	1.05	1.01	1.05	1.00
	EQL	9.19	7.41	17.33	9.22	7.36	25.12	6.25
-0.9		h=2.2	L_E=2.875	h=7.28	h=2.2	L_E=2.875	L_S=2.703	L_M=3.62
	0	370.33	371.80	370.13	371.13	373.17	372.08	373.43
	0.5	42.01	5.81	6.08	41.95	5.83	144.13	13.04
	1	4.72	2.69	3.80	4.71	2.69	49.87	1.48
	1.5	2.72	1.86	3.01	2.73	1.88	19.67	1.07
	2	1.98	1.43	2.48	1.98	1.43	7.36	1.01
	2.5	1.67	1.09	2.01	1.68	1.10	2.41	1.00
	3	1.24	1.01	2.00	1.25	1.01	1.27	1.00
	3.5	1.03	1.00	2.00	1.04	1.00	1.03	1.00
	4	1.00	1.00	2.00	1.00	1.00	1.00	1.00
	EQL	8.95	6.28	11.63	8.96	6.28	25.85	5.83



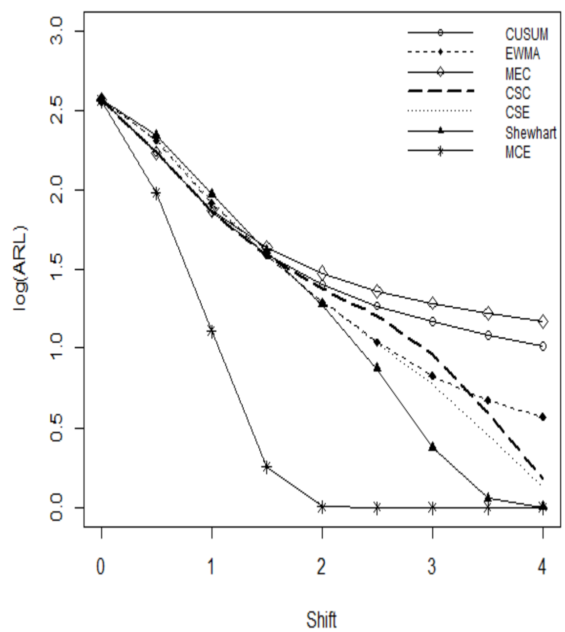
(a) $\phi = 0.25$



(b) $\phi = 0.50$

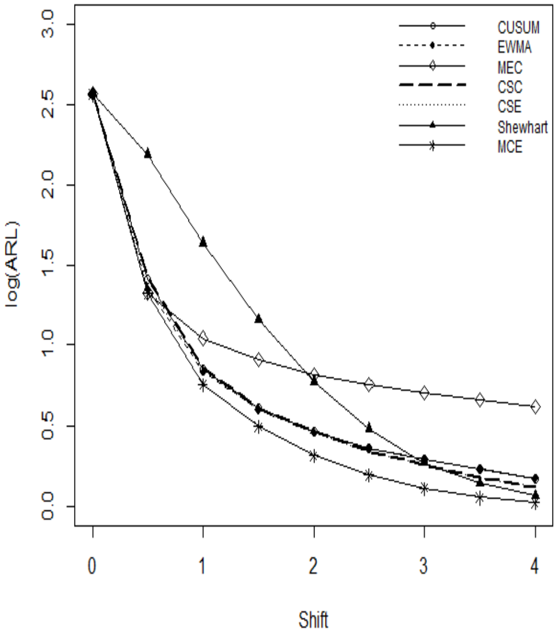


(c) $\phi = 0.75$

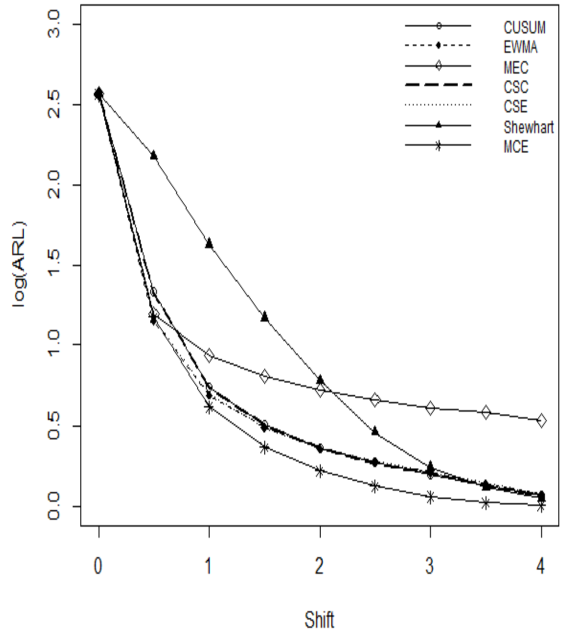


(d) $\phi = 0.90$

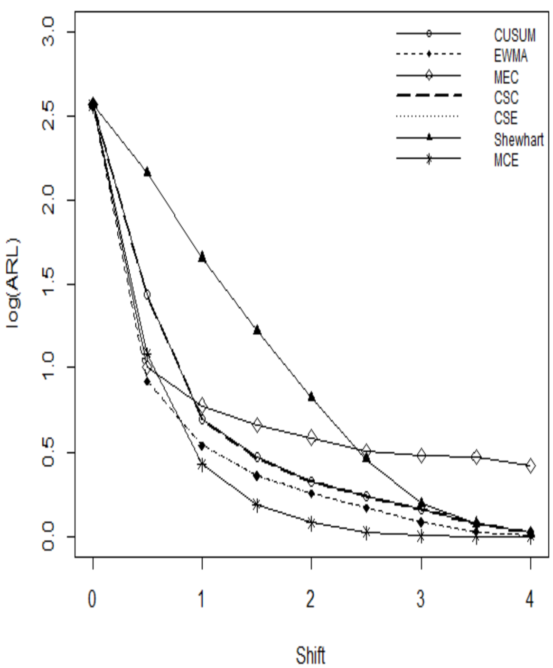
Figure 3.1: Various *ARL* curves of AR (1) process for $\phi > 0$



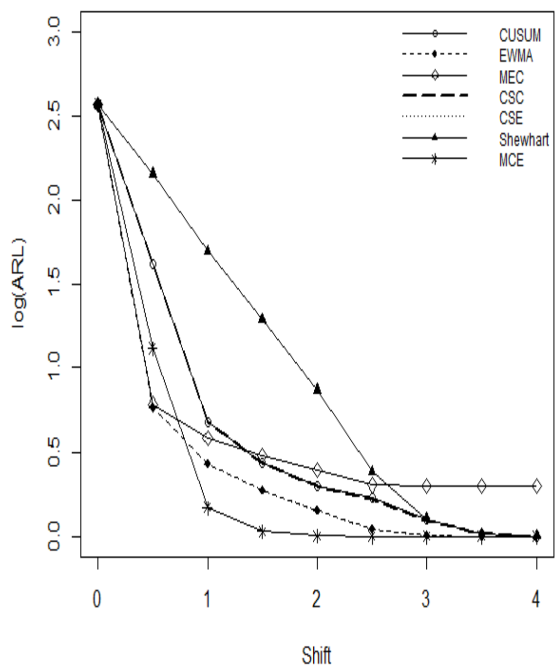
(a) $\phi = -0.25$



(b) $\phi = -0.50$



(c) $\phi = -0.75$



(d) $\phi = -0.90$

Figure 3.2: Various *ARL* curves of AR (1) process for $\phi < 0$

3.4. Performance Comparison of the Control Charts

In this section, we compare the performances of the various modified charts discussed in section 3.2 based on the *ARL* and *EQL* measures. We group the comparison discussion under uncorrelated, positively and negatively correlated processes.

3.4.1. *ARL*

Uncorrelated processes ($\phi = 0$)

An uncorrelated process has independent and random observations. It is evident from Table 3.1 that when $\phi = 0$, the MCE modified chart consistently performs better than the others for all levels of mean shifts in the process. We emphasize that the MEC, EWMA, CUSUM, CSE and CSC modified charts also perform creditably well like the MCE modified chart for small shifts whilst the CSE,CSC and the Shewhart modified charts perform quite well for large shifts especially with $\delta > 3.5$. As expected the EWMA, CUSUM, CSC and the CSE modified charts perform almost the same for medium shifts.

Positively correlated processes ($\phi > 0$)

With $\phi > 0$ in Table 3.1, the MCE modified chart is superior for small mean shifts (δ <1) but with shifts $\delta \le 0.5$ the CUSUM, CSC and MEC modified charts perform equally well for $\phi \le 0.50$. Note that the Shewhart modified chart is the worst chart for small shifts. For medium shifts, the MCE modified chart performs better than the other charts. The EWMA and CSE modified charts perform slightly better than the CUSUM and CSC modified charts for shifts $1 \le \delta \le 2.5$. However, with shifts $2.5 < \delta < 3$, the

Shewhart modified chart performs better than the MEC, EWMA, CUSUM, CSE and CSC modified charts. Overall the MEC modified chart is the worst chart for medium shifts. Again, the MCE modified chart is superior to the other charts for large shifts. However, for positively correlated processes, the Shewhart modified chart does perform better than the MEC, EWMA, CUSUM, CSE and CSC modified charts for large shifts. The MEC modified chart again is the worst chart for large mean shifts in the process.

Negatively correlated processes ($\phi < 0$)

From Table 3.2, it is noted that with $\phi \geq -0.75$ the MCE, MEC, EWMA and CSE modified charts perform equally well and slightly better than the CUSUM and CSC modified charts for small shifts. Also, when $\phi = -0.90$, the EWMA, CSE and MEC modified charts perform better for small shifts. The Shewhart modified chart is the worst chart for small shifts when the process is negatively correlated. For medium shifts, the MCE modified chart performs better than the other charts but with $\phi \geq -0.75$ the EWMA, CUSUM, CSE and CSC modified charts perform almost the same but inferior to the MCE modified chart. Lastly, the MCE modified chart performs better than the other charts for large mean shifts. Take note that for $\phi \geq -0.75$, the CUSUM, EWMA, CSC, CSE and the Shewhart modified charts almost perform the same for large shifts. With $\phi = -0.90$ all the charts with the exception of the MEC, perform almost the same as the MCE modified chart for large shifts. The performance of the MEC modified chart is worst for large mean shifts.

3.4.2. *EQL*

Consistently, the MCE modified chart has the smallest *EQL* values when the process is positively correlated when $ARL_0 = 370$ in all the charts. This is generally followed by the CSE and CSC modified charts when $\phi \leq 0.75$ and the Shewhart and EWMA modified charts when $\phi = 0.90$. The MEC modified chart has the largest *EQL* values for autocorrelated processes. These findings show that the MEC and the MCE are the worst and the best performing modified charts respectively when $\phi \geq 0$. As expected the MCE modified chart has the smallest *EQL* values when $\phi < 0$ as compared to the others. This is also followed by CSE and CSC when $\phi = -0.25$ and the CSE and EWMA modified charts when $\phi \leq -0.50$. These results show that the MCE modified chart has the best performance capability when $\phi < 0$ whilst the MEC and Shewhart modified charts are the worst performing charts for $\phi = -0.25$ and $\phi \leq -0.50$ respectively.

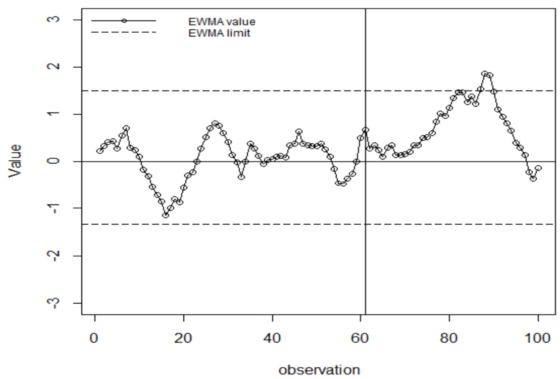
3.5. Illustrative Example

In this section, we demonstrate how the charts perform with respect to autocorrelated data using a simulated dataset. In this example we simulated 100 observations from an AR (1) model with $\phi = 0.5$. The first 60 observations represent the in-control state of the process. At observation 61, a shift of $0.5\sigma_X$ is introduced in the second half of the data. The parameters for the charts are selected in order to generate an in-control *ARL*=370 when $\phi = 0.5$. Figures 3.3(a)-(e) displays the graphical outputs from the charts and Table

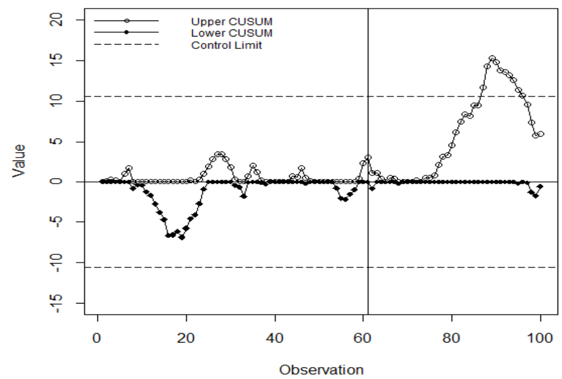
3.3 displays the out-of-control points for the data. Due to lack of space, the first 30 values of the statistics are displayed in Table 3.4.

Table 3.3: Out-of-control points in the charts

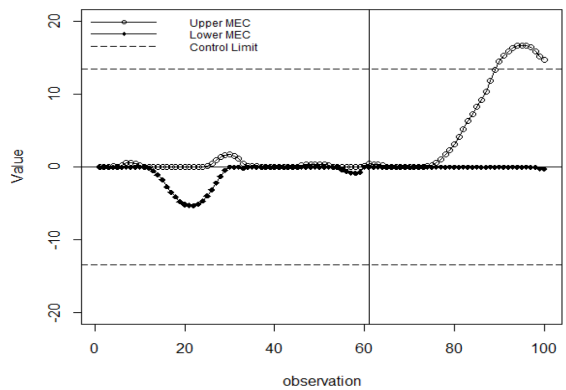
Charts	Out of control points
Shewhart	88
EWMA	87,88,89
CUSUM	87,88,89,90,91,92,93,94,95, 96
MEC	90,91,92,93,94,95,96,97,98,99,100
MCE	88,89,90,91,92,93,94,95,96,97,98,99



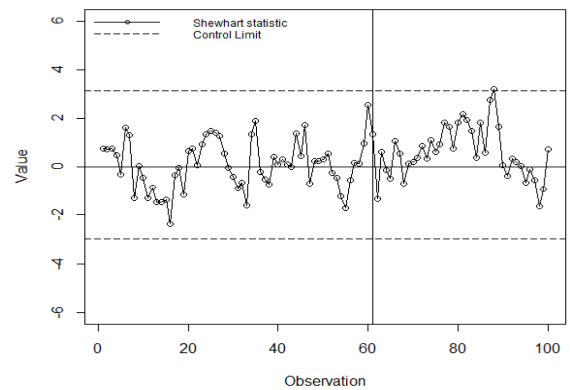
(a) EWMA chart



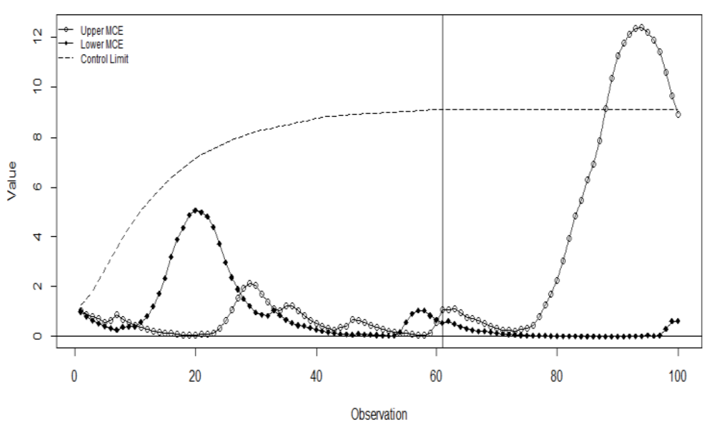
(b) CUSUM chart



(c) MEC



(d) Shewhart chart



(e) MCE chart

Figure 3.3: Modified charts for the simulated data

It is clear from Figure 3.3(a-e) that when the shift is introduced into the process, the MCE, MEC, CUSUM, EWMA and Shewhart modified charts detected 12, 11,10,3 and 1 out of control points respectively. Additionally, it is suspected that observation 100 in the MCE modified chart may be out-of-control. It is evident that from the demonstration that the MCE and MEC modified charts have the best detection ability because they detected the most signals and the Shewhart modified chart performed worst. These conclusions are consistent with the results in Section 3.4 concerning positively correlated data for the magnitude of shift considered.

Table 3.4: Values of the statistics (first 30 out of 100)

Observation	Shewhart	EWMA	CUSUM		MEC		MCE	
	W_t	Y_t	C_t^+	C_t^-	MEC_t^+	MEC_t^-	MCE_t^+	MCE_t^-
1	0.76	0.22	0.00	0.00	0.00	0.00	1.03	1.00
2	0.74	0.33	0.14	0.00	0.00	0.00	0.88	0.80
3	0.77	0.42	0.30	0.00	0.06	0.00	0.80	0.64
4	0.50	0.43	0.19	0.00	0.14	0.00	0.71	0.51
5	-0.31	0.28	0.00	0.00	0.08	0.00	0.57	0.41
6	1.63	0.55	1.03	0.00	0.28	0.00	0.66	0.33
7	1.31	0.71	1.74	0.00	0.63	0.00	0.88	0.26
8	-1.31	0.30	0.00	0.88	0.58	0.00	0.70	0.39
9	0.05	0.25	0.00	0.41	0.48	0.00	0.56	0.39
10	-0.45	0.11	0.00	0.44	0.24	0.00	0.45	0.40
11	-1.29	-0.17	0.00	1.30	0.00	0.00	0.36	0.58
12	-0.86	-0.31	0.00	1.74	0.00	0.14	0.29	0.81
13	-1.46	-0.54	0.00	2.78	0.00	0.50	0.23	1.21
14	-1.46	-0.72	0.00	3.81	0.00	1.05	0.18	1.73
15	-1.35	-0.85	0.00	4.74	0.00	1.73	0.15	2.33
16	-2.36	-1.15	0.00	6.68	0.00	2.71	0.12	3.20
17	-0.34	-0.99	0.00	6.59	0.00	3.52	0.09	3.88
18	-0.02	-0.80	0.00	6.19	0.00	4.15	0.08	4.34
19	-1.15	-0.87	0.00	6.91	0.00	4.84	0.06	4.85
20	0.68	-0.56	0.08	5.80	0.00	5.22	0.06	5.04
21	0.77	-0.29	0.24	4.61	0.00	5.34	0.10	4.96
22	0.09	-0.22	0.00	4.10	0.00	5.39	0.08	4.79
23	0.94	0.01	0.33	2.73	0.00	5.20	0.13	4.38
24	1.35	0.28	1.08	0.96	0.00	4.75	0.32	3.69
25	1.50	0.52	1.97	0.00	0.17	4.05	0.65	2.95
26	1.44	0.71	2.80	0.00	0.53	3.17	1.08	2.36
27	1.27	0.82	3.47	0.00	1.00	2.18	1.56	1.89
28	0.57	0.77	3.44	0.00	1.41	1.23	1.93	1.51
29	-0.02	0.61	2.82	0.00	1.68	0.45	2.11	1.21
30	-0.39	0.41	1.82	0.00	1.74	0.00	2.05	0.97

3.6. Conclusions

The impact of autocorrelation on control charts is of major concern in process monitoring. The presence of autocorrelation can make the charts malfunction by producing frequent false alarms. To overcome this setback, modified charts can be used to control the problems associated with serial correlation. The modified charts are implemented by adjusting the standard control limits of the charts. In this study, we proposed the MEC and MCE modified charts as alternative charts to the existing (Shewhart, CUSUM, EWMA, CSE and CSC) modified charts.

For the AR (1) process model, it is evident that the MCE modified chart is superior in detecting small, medium and large mean shifts better than the existing modified charts for correlated data. Furthermore, the comparison showed that the MEC modified chart is efficient for monitoring small mean shifts (usually $\delta \leq 0.5$) slightly better than the existing charts for positively correlated processes. For future research, the performance of the MCE and MEC modified charts can be extended to other autocorrelated processes that can be fitted with a linear trend AR (1), ARMA (1, 1) and AR (2) models.

CHAPTER 4

OPTIMIZATION DESIGN OF THE CUSUM AND EWMA CHARTS FOR AUTOCORRELATED PROCESSES

4.1. Introduction

Statistical process control (SPC) is an approach which uses statistical methods and tools to analyse and monitor the variability in the process to ensure it achieves stability and improve the performance of the process (Montgomery (2009)). A control chart is an example of a tool used in SPC. The Shewhart, cumulative sum (CUSUM) and the exponentially weighted moving average (EWMA) charts are widely used to detect shifts of process parameters. Control charts are used based on the assumptions that observations from the process are independently and normally distributed. In the event of a violation of these assumptions, the charts malfunction and lead to producing frequent false alarm signals even when the process is stable (Alwan and Roberts (1988), Harris and Ross (1991), Wieringa (1999), Zhang (2000), Montgomery (2009)).

In the presence of serial correlation, residual and modified charts are widely used to account for the autocorrelation in the process. In residual charts, the traditional control charts are applied on the residuals obtained from fitting a time series model to the correlated data. Works by Alwan and Roberts (1988), Harris and Ross (1991), Wardel *et al*. (1992), Wieringa (1999), Lu and Reynolds (1999), Zhang (2000) and others applied residual charts on autocorrelated data to effectively monitor the process. However, in the modified charts the standard control limits of the conventional charts are adjusted when they are applied directly on the correlated data (Vasilopoulos and Stamboulis (1978), Schmid (1995), Kramer and Schmid (1997)).

The traditional CUSUM and EWMA charts are modified into the CUSUM and EWMA modified charts or chart of observations respectively in the presence of serial correlation to handle the problems associated with the correlation (Vanbrackle and Reynolds (1997), Wieringa (1999), Lu and Reynolds (1999, 2001)). The variance of the underlying time series model is used instead of the independent observations (Knoth and Schmid (2002)). These charts have design parameters in their formulation which are chosen during the implementation of the charts to monitor the process. It is worth pointing out that in reality the magnitude of the shift is unknown or uncertain and therefore the performance of the charts are poor when the choice of parameters for the assumed shift are considerably different to that of the actual shift (Ryu *et al*.(2010)). Hence to overcome this problem the charts are optimized by selecting design parameters to improve the sensitivity of the charts to shifts in the mean based on certain performance measures such as *ARL*, *EQL* and *PCI*.

Several researchers have conducted studies into the optimization of various control charts using different performance criteria. For independent and normal processes, Aparisi and Garcia-Diaz (2004, 2007) used a genetic algorithm procedure to optimally design the univariate and multivariate EWMA charts and also used the same procedure to determine the parameters of the EWMA chart for different control regions by computing the *ARL* values. Chen and Chen (2007) imposed an in-control (ARL_0)

value on the process and determined the parameters in the EWMA and CUSUM charts that optimize the charts by minimizing the *EQL* value. On their part Wu *et al*. (2008) optimized the designs of the combined Shewhart-CUSUM (CSC) control chart to determine values of the parameters (reference, decision interval and upper control limit values) that optimize the chart by fixing ARL_0 value and minimizing the *EQL* value. Following their work on the CSC chart, Wu *et al*. (2010) again worked on optimizing the performance of the 2 CUSUM schemes for monitoring shifts in the mean and variance of the chart by minimizing the average extra quadratic loss (*AEQL*) criterion by keeping the in-control average time to signal (ATS_0) criterion for the in control process constant to determine the four parameters in the chart. Ryu *et al*. (2010) worked on the optimisation design of various CUSUM charts with an unknown mean shift using the expected weighted run length (*EWRL*) performance criterion.

In the presence of serially correlated data, Singh and Prajapati (2013) studied the performance of the EWMA and CUSUM charts. In the work, they suggest parameters that should be paired with k=0.5 and $\lambda = 0.2$ in the CUSUM and EWMA charts respectively in order to optimize them. Works by authors such as Lu and Reynolds (1999, 2001) and Knoth and Schmid (2004) determined the optimal parameters in the CUSUM and/or EWMA charts with respect to ARL_1 using different process models for correlated data. Cheng and Thaga (2005) and Thaga and Yadavalli (2007) conducted similar works using the MAX-EWMA and MAX-CUSUM charts respectively.

In this chapter, we analyse and optimize the performance of the CUSUM and EWMA modified charts for location using correlated observations that can be modelled with an AR (1) process model. The structures of these two modified charts are discussed in Section 3.2. Only positive correlations $\phi_1 = \{0, 0.1, 0.15, ...0.85, 0.9, 0.95\}$ are considered in this work because their presence is more likely in manufacturing operations (Woodall and Faltin (1993)). For each autocorrelation coefficient, we will determine all the different combinations of the design parameters (with a step increment of 0.01) in the charts that yield a specific ARL_0 value using the exhaustive search algorithm. Furthermore, we determine the parameters that optimize the design of the charts in terms of the *EQL* and *PCI*.

The rest of the chapter is structured as follows: Next Section talks about the optimization designs of the CUSUM and EWMA modified charts. Section 4.3 talks about the comparison for these modified charts. An Illustrative example is organised in Section 4.4 to demonstrate how the charts perform. Conclusions are discussed in Section 4.5.

4.2. Optimization Design of the Charts

Different optimization techniques exist that can be used to determine the optimal quantities of a system or process in many fields. These techniques include simulated annealing (Haddock and Mittenthal (1992) and Goffe *et al*. (1994)) and genetic algorithm (Painton and Campbell (1995), Grefenstette (1986), Aparisi and Garcia-Diaz (2004)). Usage of the estimated optimal values enhances the performance ability of the system to achieve maximum output. In this study, we use the exhaustive search optimization procedure to determine the optimal values in EWMA and CUSUM modified charts. Exhaustive search is a problem solving approach which is used to find all possible combinations or candidates that satisfy all given constraints or the problem condition (Nievergelt (2000)). The drawback of this procedure is that it takes a lot of time to complete because it involves a lot of computations especially when design parameters are many. The EWMA and CUSUM modified charts have parameters (λ, L) and (k , h) respectively. The optimization procedure is summarised as follows;

Objective function: Determine pairs of parameters that minimize *EQL* and *PCI*. The pairs that yield the smallest ARL_1 value at each level of shift can also be determined.

Set Constraint condition: $ARL_0 - b \leq ARL_0 \leq ARL_0 + b$

Variables in the model: λ, L

Independent variable: λ

Dependent variable: L

Where *b* represents a permissible deviation from the expected ARL_0 value.

The exhaustive search procedure for the design model is as follows;

1. Define range for the variables (i.e. $L_{\min} \leq L \leq L_{\max}$ and $0 < \lambda_{\min} < \lambda < \lambda_{\max} \leq 1$).
2. Begin the process with a selected λ value (e.g. $\lambda_{\max}$)
3. Select L value (e.g. $L_{\max}$)
4. Generate W_t from equation (4.3) and perform 10,000 simulations to determine if the combination of these initial parameters satisfy the constraint condition when the shift $\delta = 0$.

5. If the constraint condition is satisfied then decrease the initial λ parameter by 0.01 to get λ_{new} and repeat the process again from step 2 but if the constraint is not met only decrease the initial L value by 0.01 to get L_{new} and repeat from step 3 till a value of L is found which combines with the initial λ value to satisfy the constraint condition.
6. Estimate the *EQL* values for all the combinations of the parameters that satisfy the constraint condition and subsequently determine the particular combination of the parameters that minimizes the *EQL* value for the given shifted range of interest (e.g. [0, 3] with increment of 0.25). Determine the *PCI* and combinations that give the smallest ARL_1 value for each shift value.

Note that a similar procedure is used to determine the optimal values (k, h) for the CUSUM modified chart within the range ($k_{\min}, k_{\max}$ and $h_{\min}, h_{\max}$). $k_{\min}$ and $h_{\min}$ are fixed at 0.01. The optimization procedures are computerized using R 3.1.2 software. Due to lack of space we have provided two examples of the various combinations of parameters that yield the desired $ARL_0 = 370$ with the respective *EQL* values when $\phi_1 = 0.5$ in the EWMA and CUSUM modified charts in Tables 4.1 and 4.2. Figures 4.1 and 4.2 present the graphical displays of the combination of parameters in EWMA and CUSUM modified charts that yield the ARL_0 value for some selected ϕ_1 values.

Table 4.1: Parameters in the EWMA modified chart when $\phi_1 = 0.5$ and $ARL_0 = 370$

λ	L	EQL	PCI
0.98	2.97	34.79	1.46608
0.97	2.97	34.83	1.46776
0.95	2.97	34.59	1.45765
0.94	2.97	34.37	1.44838
0.93	2.97	34.42	1.45048
0.92	2.97	34.34	1.44711
0.9	2.97	33.98	1.43194
0.88	2.97	34.01	1.43321
0.87	2.97	33.92	1.42941
0.86	2.96	33.12	1.3957
0.84	2.96	33.04	1.39233
0.81	2.96	32.89	1.38601
0.8	2.95	32.11	1.35314
0.79	2.95	32.05	1.35061
0.78	2.95	32.04	1.35019
0.77	2.95	31.93	1.34555
0.76	2.95	31.74	1.33755
0.74	2.95	31.63	1.33291
0.73	2.94	31.02	1.30721
0.72	2.94	31.03	1.30763
0.71	2.94	31.09	1.31016
0.7	2.94	30.93	1.30341
0.69	2.94	30.82	1.29878
0.68	2.93	30.32	1.27771
0.66	2.93	30.29	1.27644
0.65	2.93	30.1	1.26844
0.64	2.92	29.8	1.25579
0.63	2.92	29.63	1.24863
0.62	2.92	29.5	1.24315
0.6	2.92	29.35	1.23683
0.59	2.91	29.14	1.22798
0.58	2.91	28.9	1.21787
0.57	2.91	28.82	1.2145
0.56	2.9	28.36	1.19511
0.55	2.9	28.49	1.20059
0.54	2.9	28.37	1.19553
0.53	2.89	27.91	1.17615

λ	L	EQL	PCI
0.51	2.89	27.83	1.17278
0.5	2.89	27.93	1.17699
0.48	2.88	27.35	1.15255
0.46	2.87	27.07	1.14075
0.45	2.87	27.01	1.13822
0.44	2.87	26.86	1.1319
0.43	2.86	26.58	1.1201
0.41	2.85	26.23	1.10535
0.4	2.85	26.22	1.10493
0.37	2.84	26.08	1.09903
0.36	2.83	25.77	1.08597
0.34	2.82	25.39	1.06995
0.33	2.82	25.34	1.06785
0.3	2.8	24.95	1.05141
0.29	2.79	24.64	1.03835
0.28	2.78	24.47	1.03118
0.27	2.78	24.57	1.0354
0.24	2.75	24.18	1.01896
0.23	2.75	24.24	1.02149
0.21	2.73	24.06	1.01391
0.19	2.71	24.04	1.01306
0.18	2.69	23.77	1.00169
0.17	2.68	23.76	1.00126
0.16	**2.66**	**23.73**	**1**
0.15	2.65	23.85	1.00506
0.14	2.64	24.01	1.0118
0.13	2.62	24.02	1.01222
0.12	2.61	24.27	1.02276
0.11	2.58	24.25	1.02191
0.1	2.55	24.43	1.0295
0.09	2.53	24.79	1.04467
0.08	2.5	25.16	1.06026
0.07	2.46	25.56	1.07712
0.06	2.42	26.27	1.10704
0.05	2.36	26.97	1.13654
0.04	2.29	28.16	1.18668
0.03	2.19	29.84	1.25748
0.01	1.74	36.81	1.5512

Table 4.2: Parameters in the CUSUM modified chart when $\phi_1 = 0.5$ and $ARL_0 = 370$

k	*h*	*EQL*	*PCI*
0.01	41.14	64.59	2.70477
0.02	38.89	61.56	2.57789
0.03	36.96	59.07	2.47362
0.04	35.19	56.68	2.37353
0.05	33.58	54.59	2.28601
0.06	32.14	52.68	2.20603
0.07	30.8	50.93	2.13275
0.08	29.49	49.25	2.0624
0.09	28.33	47.65	1.99539
0.1	27.25	46.29	1.93844
0.11	26.28	45.01	1.88484
0.12	25.41	43.86	1.83668
0.13	24.42	42.58	1.78308
0.14	23.69	41.72	1.74707
0.15	22.95	40.72	1.70519
0.16	22.24	39.83	1.66792
0.17	21.5	38.81	1.62521
0.18	20.87	38.11	1.5959
0.19	20.29	37.41	1.56658
0.2	19.73	36.66	1.53518
0.21	19.24	36.13	1.51298
0.22	18.64	35.42	1.48325
0.23	18.24	34.93	1.46273
0.24	17.78	34.31	1.43677
0.25	17.27	33.73	1.41248
0.26	16.89	33.32	1.39531
0.27	16.46	32.8	1.37353
0.28	16.07	32.34	1.35427
0.29	15.69	31.89	1.33543
0.3	15.33	31.52	1.31993
0.31	14.98	31.13	1.3036
0.32	14.64	30.71	1.28601
0.33	14.33	30.36	1.27136
0.34	14.11	30.2	1.26466
*	*	*	*

k	*h*	*EQL*	*PCI*
0.83	6.19	23.905	1.00105
0.84	6.11	24.02	1.00586
0.85	6.01	23.95	1.00293
0.86	5.95	24.02	1.00586
0.87	5.85	23.904	1.00101
0.88	**5.76**	**23.88**	1
0.89	5.71	24.04	1.0067
0.9	5.61	23.96	1.00335
0.91	5.54	23.97	1.00377
0.92	5.48	23.99	1.00461
0.93	5.39	23.93	1.00209
0.94	5.31	23.92	1.00168
0.95	5.26	24.06	1.00754
0.96	5.16	23.99	1.00461
0.97	5.12	24.04	1.0067
0.98	5.03	23.98	1.00419
0.99	4.98	24.06	1.00754
*	*	*	*
2.75	0.23	35.09	1.46943
2.76	0.22	35.14	1.47152
2.79	0.19	35.38	1.48157
2.81	0.17	35.47	1.48534
2.82	0.16	35.53	1.48786
2.83	0.15	35.56	1.48911
2.84	0.14	35.66	1.4933
2.86	0.12	35.67	1.49372
2.87	0.11	35.63	1.49204
2.88	0.1	35.65	1.49288
2.89	0.09	35.63	1.49204
2.9	0.08	35.61	1.49121
2.92	0.06	35.77	1.49791
2.93	0.05	35.7	1.49497
2.95	0.03	35.89	1.50293
2.96	0.02	35.76	1.49749
2.97	0.01	35.87	1.50209

* The *k* values in the intervals [0.35, 0.82] and [1, 2.74] with their corresponding *h*, *EQL* and *PCI* values have been omitted for the sake of brevity.

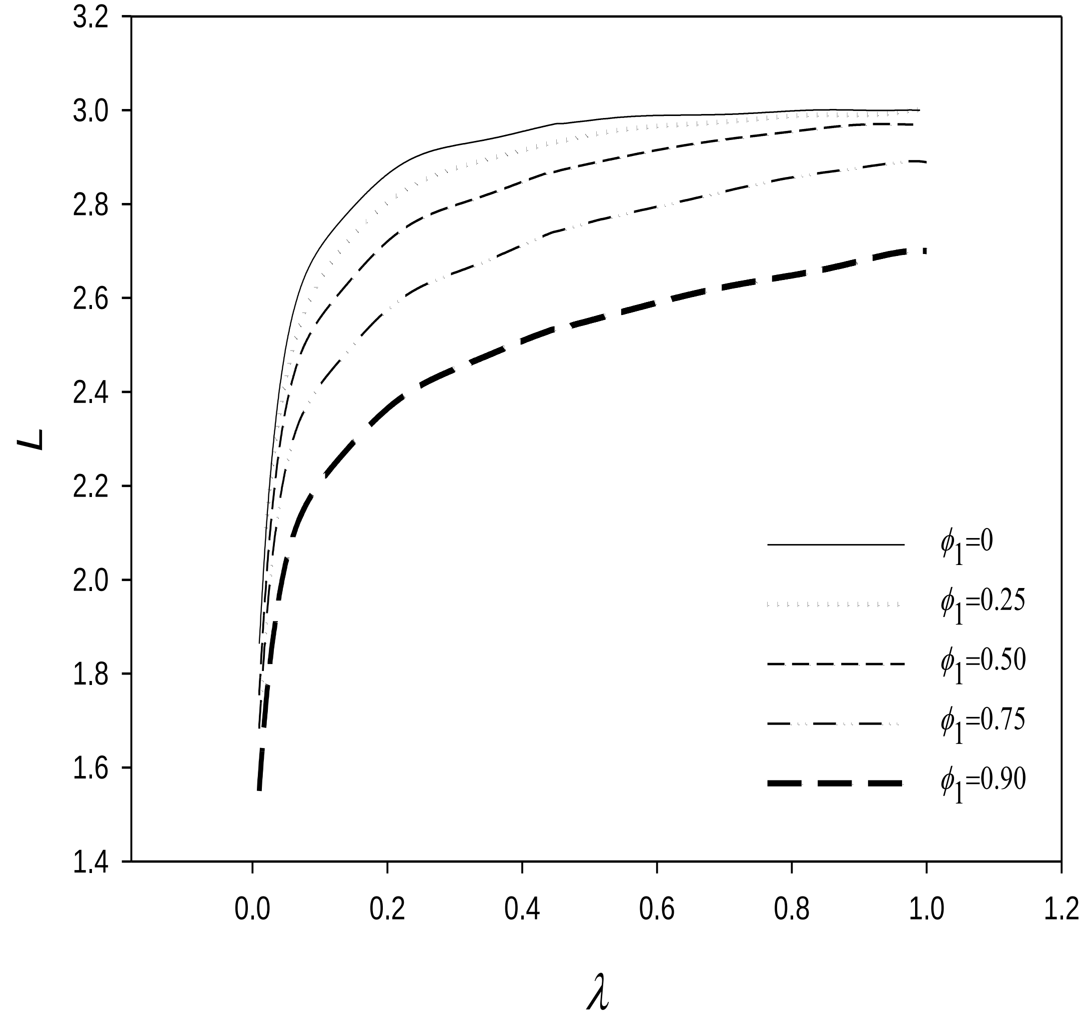


Figure 4.1: Plots of L and λ for the EWMA modified chart for different values of ϕ_1

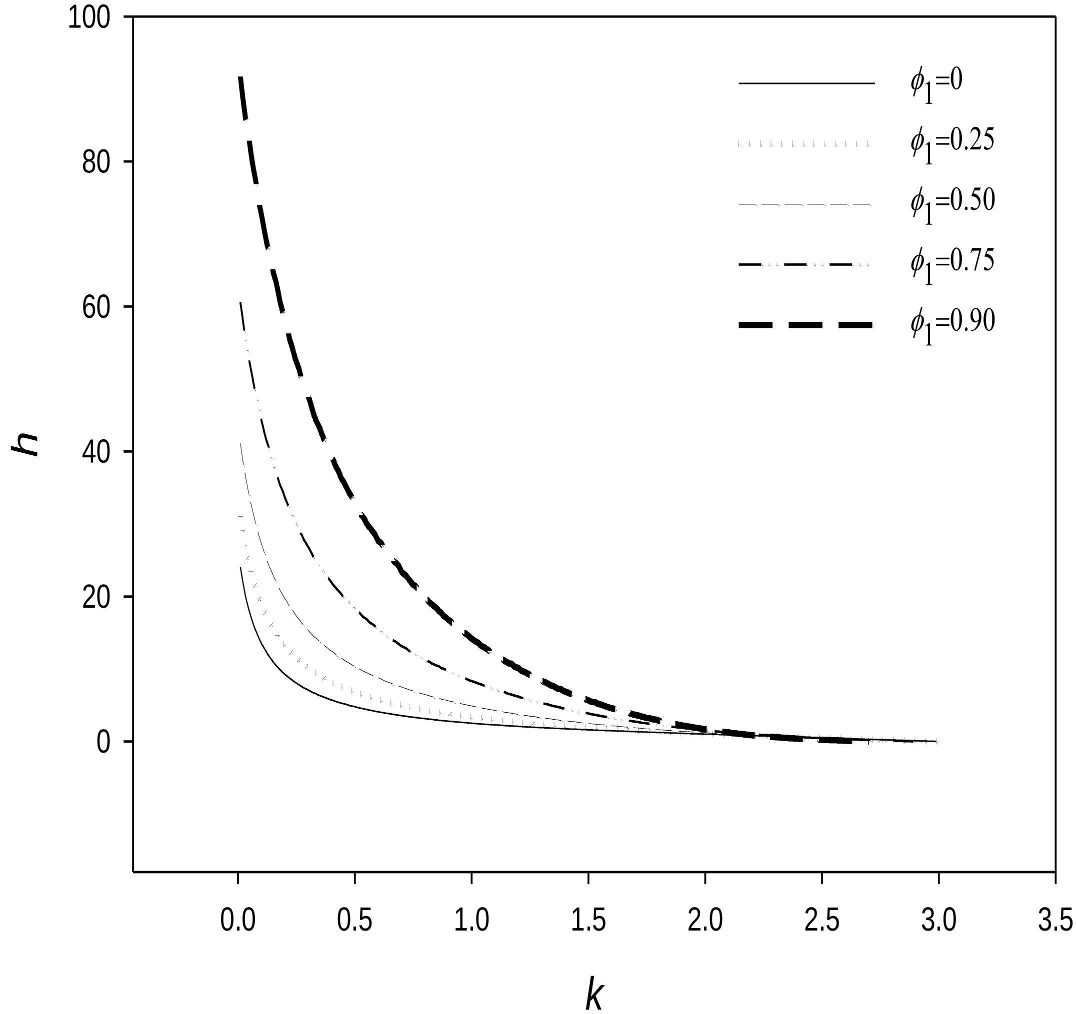


Figure 4.2: Plots *k* and *h* for the CUSUM modified chart for different values of ϕ_1

Table 4.3: Optimal parameters in the Modified charts with respect to *EQL* when $ARL_0 = 370$

	EWMA			CUSUM		
ϕ_1	λ	L	EQL_{opt}	k	h	EQL_{opt}
0	0.26	2.9	12.29353	0.71	3.51	12.51706
0.05	0.23	2.87	12.96398	0.68	3.9	13.23578
0.1	0.21	2.84	13.66856	0.69	4.11	14.00597
0.15	0.2	2.82	14.56917	0.75	4.05	14.87002
0.2	0.21	2.82	15.56732	0.78	4.17	15.77188
0.25	0.24	2.83	16.58385	0.8	4.35	16.81019
0.3	0.2	2.78	17.64995	0.83	4.48	17.86046
0.35	0.17	2.73	18.93886	0.81	4.96	19.11951
0.4	0.16	2.7	20.29619	0.84	5.14	20.44552
0.45	0.18	2.71	21.91483	0.86	5.45	22.12922
0.5	0.16	2.66	23.73461	0.88	5.76	23.87732
0.55	0.15	2.63	25.96159	0.95	5.73	25.90207
0.6	0.17	2.63	28.35096	1.02	5.72	28.30858
0.65	0.12	2.52	31.36521	1.03	6.23	31.0766
0.7	0.14	2.52	34.81751	1.11	6.22	34.75162
0.75	0.15	2.5	39.68616	1.09	7.27	39.0063
0.8	0.12	2.4	45.6696	1.22	6.84	44.58356
0.85	0.2	2.45	54.21289	1.42	5.66	52.36406
0.9	1	2.7	62.74785	2.37	0.43	62.0362
0.95	1	2.49	73.17253	2.39	0.1	71.83316

With all the different combinations of parameters that yield the desired ARL_0 for each ϕ_1, the *EQL* values are estimated over the shift range $\delta \in [0,3]$ with a step shift of 0.25. From Table 4.1 with λ=0.16 and L=2.66, the EWMA modified chart has the smallest *EQL* (EQL_{opt}) value of 23.73 whilst from Table 4.2 with k = 0.88 and h=5.76, the CUSUM modified chart has the smallest *EQL* (EQL_{opt}) value of 23.88 when $\phi_1 = 0.5$ in the charts. Therefore these charts have better performance ability over the shift range using the optimal parameters compared to the other parameters. The optimal parameters for the various ϕ_1 values under consideration in this work are also determined. It is evident from Tables 4.1 and 4.2 that the optimal parameters possess the smallest *PCI* values. This trend is observed for all the optimal parameters for the different correlation parameters considered. This indicates that charts with optimal parameters have better relative performance efficiency. Table 4.3 displays the optimal parameters for each chart when $ARL_0 = 370$. We observe in Table 4.3 that λ values between [0.15, 0.25] are ideal for correlated processes with $\phi_1 \leq 0.85$ and λ=1 works well for highly correlated processes usually for $\phi_1 > 0.85$ in the EWMA modified chart to minimize *EQL*. Similarly, *k* values between [0.65, 0.85], [0.90, 1.20] and [1.40, 2.40] work well for $\phi_1 \leq 0.5$, $0.5 < \phi_1 < 0.85$ and $\phi_1 \geq 0.85$ respectively in the CUSUM modified chart for the range of shift, step shift and *ARL* considered.

Having a list of design parameters that satisfy the constraint function of $ARL_0 = 370$ for different ϕ_1 values, the combinations of parameters that yield the smallest ARL_1 value for each shift can be equally determined. These parameters also optimize the performance of the charts with respect to ARL_1. Tables 4.4 and 4.5 display the optimal parameters for the EWMA and CUSUM modified charts for a selected few autocorrelation values. As expected small values of λ are appropriate for detecting small shifts and large values for monitoring large shifts in the EWMA modified chart. Similarly, small values of *k* are ideal for detecting small shifts whilst large values of *k* are suitable for detecting large shifts for ϕ_1.

Table 4.4: Optimal parameters in the EWMA Modified charts with respect to ARL_1

$ARL_0 = 370$														
		δ												
ϕ	Parameter	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	$EQL(_{ARL})$
0	λ	0.02	0.05	0.1	0.14	0.18	0.26	0.28	0.4	0.45	0.53	0.66	0.72	
	L	2.14	2.5	2.7	2.79	2.84	2.9	2.91	2.96	2.97	2.98	2.99	2.99	
	ARL_1	67.4122	26.5141	14.6216	9.4966	6.8237	5.129	4.0896	3.3231	2.7897	2.3701	2.0362	1.7637	10.73779
0.25	λ	0.02	0.03	0.06	0.1	0.14	0.24	0.27	0.36	0.42	0.48	0.72	0.75	
	L	2.1	2.26	2.49	2.64	2.72	2.83	2.86	2.9	2.92	2.94	2.98	2.98	
	ARL_1	92.0232	37.6974	20.9602	13.5764	9.582	7.2084	5.5777	4.4499	3.6284	3.0349	2.5071	2.0746	14.35391
0.5	λ	0.01	0.03	0.04	0.07	0.08	0.16	0.18	0.27	0.36	0.51	0.98	0.94	
	L	1.74	2.19	2.29	2.46	2.5	2.66	2.69	2.78	2.83	2.89	2.97	2.97	
	ARL_1	125.164	55.7238	31.1773	20.6866	14.6159	10.7117	8.1704	6.4784	5.1456	4.0829	3.2228	2.4188	20.3897
0.75	λ	0.01	0.01	0.02	0.03	0.07	0.07	0.15	0.21	0.3	1	1	1	
	L	1.66	1.66	1.92	2.07	2.32	2.32	2.5	2.58	2.65	2.89	2.89	2.89	
	ARL_1	191.074	90.7624	54.5183	36.181	25.7774	19.0922	14.5592	11.145	8.4487	5.9021	4.0518	2.7378	33.042
0.9	λ	0.01	0.01	0.01	0.03	0.03	0.06	0.2	0.82	1	0.99	1	1	
	L	1.52	1.52	1.52	1.89	1.89	2.07	2.36	2.65	2.7	2.7	2.7	2.7	
	ARL_1	270.709	154.609	97.6226	67.8122	48.6811	35.443	26.6425	18.8593	12.1101	7.4359	4.3252	2.3765	53.52688

Table 4.5: Optimal parameters in the CUSUM Modified charts with respect to ARL_1

ARL_0=370														
		δ												
ϕ	Parameter	0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3	$EQL(_{ARL})$
0	k	0.12	0.24	0.35	0.51	0.63	0.78	0.94	0.99	1.06	1.26	1.37	1.51	
	h	12.43	8.26	6.33	4.7	3.91	3.22	2.68	2.54	2.37	1.97	1.79	1.59	
	ARL_1	74.2722	28.8884	15.5052	9.8983	6.8957	5.1313	3.9907	3.2156	2.6876	2.2647	1.9551	1.7042	10.67522
0.25	k	0.1	0.25	0.37	0.52	0.61	0.81	1	1.09	1.33	1.59	1.68	1.79	
	h	18.92	11.52	8.71	6.6	5.71	4.29	3.36	3.02	2.26	1.69	1.53	1.36	
	ARL_1	103.125	41.0013	22.3863	14.184	9.7917	7.191	5.4661	4.2685	3.4418	2.774	2.2969	1.9311	14.13227
0.5	k	0.14	0.22	0.39	0.49	0.7	0.83	0.93	1.22	1.46	1.7	1.78	2.1	
	h	23.69	18.64	12.68	10.52	7.51	6.19	5.39	3.63	2.61	1.88	1.66	1.03	
	ARL_1	141.868	60.7479	33.9213	21.666	14.901	10.7446	8.0598	6.1572	4.7607	3.7144	2.9117	2.2746	20.09516
0.75	k	0.12	0.27	0.39	0.47	0.73	0.92	1.04	1.19	1.76	2.05	2.44	2.49	
	h	41.69	28.65	22.38	19.39	12.56	9.41	7.88	6.27	2.46	1.36	0.54	0.46	
	ARL_1	208.069	102.47	59.399	38.984	26.856	19.449	14.3059	10.5454	7.7635	5.5535	3.8776	2.6865	33.17994
0.9	k	0.23	0.27	0.43	0.6	0.78	1.15	1.31	1.62	2.54	2.54	2.61	2.62	
	h	54.55	50.26	37.29	27.67	20.5	10.97	8.24	4.42	0.17	0.17	0.09	0.08	
	ARL_1	281.352	169.561	107.116	71.746	49.9363	35.9065	25.8057	18.0453	11.8938	7.253	4.0431	2.284	54.03309

Furthermore with a list of the ARL_1 values for each ϕ_1, the *EQL* value we denote by EQL_{ARL_1} can also be estimated. This value is obtained by using the ARL_1 values in the computation of the *EQL* and as expected it is smaller than the EQL_{opt} and EQL_{con} for each ϕ_1 considered in the charts. EQL_{con} is the *EQL* obtained by using the conventional parameters of the charts. That is $EQL_{ARL_1} < EQL_{opt} < EQL_{con}$.

Example with $\phi_1 = 0.5$;

The EWMA chart has $EQL_{ARL_1} = 20.39$; $EQL_{opt} = 23.73$; $EQL_{con} = 24.07$.

The CUSUM chart has $EQL_{ARL_1} = 20.10$; $EQL_{opt} = 23.88$; $EQL_{opt} = 26.43$.

4.3. Comparison of the CUSUM and EWMA Modified Charts

In this section, we investigate the performances of the charts based on the ARL, EQL and PCI measures. Conventionally, λ=0.2 and *k*=0.5 are widely used values in the EWMA and CUSUM charts respectively because they make the charts very sensitive for small to medium shifts. We use these parameters in the conventional EWMA and CUSUM charts whilst the optimal parameters from Table 4.3 are used in the optimized EWMA and CUSUM charts. We compare the charts using the conventional and optimized parameters for some selected values of ϕ_1. Table 4.6 displays the ARL comparison of the conventional and optimized charts.

Table 4.6: *ARL* comparison of the charts

Chart	Parameters	ϕ_1	δ												*EQL*	*PCI*
			0.25	0.5	0.75	1	1.25	1.5	1.75	2	2.25	2.5	2.75	3		
con EWMA (λ, L)	(0.2,2.86)	0	120.611	36.463	16.377	9.787	6.866	5.238	4.206	3.611	3.125	2.778	2.515	2.311	12.341	1.004
opt EWMA	(0.26,2.9)		136.918	42.179	17.983	10.388	6.928	5.190	4.146	3.418	2.974	2.620	2.368	2.157	12.294	1.000
con CUSUM (k,h)	(0.5,4.79)		123.016	35.479	16.271	9.965	7.071	5.530	4.555	3.855	3.382	2.994	2.721	2.487	13.012	1.058
opt CUSUM	(0.71,3.51)		155.448	46.357	19.265	10.657	6.992	5.145	4.119	3.426	2.966	2.607	2.340	2.130	12.517	1.018
con EWMA (λ, L)	(0.2,2.80)	0.25	164.919	56.518	25.819	14.709	9.758	7.229	5.723	4.691	3.980	3.468	3.091	2.800	16.606	1.001
opt EWMA	(0.24,2.83)		171.860	60.732	27.920	15.475	9.981	7.155	5.620	4.560	3.838	3.340	2.963	2.676	16.584	1.000
con CUSUM (k,h)	(0.5,6.84)		146.701	48.107	23.130	14.336	10.061	7.804	6.375	5.391	4.653	4.123	3.693	3.375	17.945	1.082
opt CUSUM	(0.8,4.35)		186.122	66.861	28.858	15.642	10.072	7.250	5.596	4.542	3.825	3.329	2.912	2.656	16.810	1.014
con EWMA (λ, L)	(0.2,2.72)	0.5	213.455	90.091	42.919	24.164	15.628	11.139	8.341	6.648	5.443	4.621	4.034	3.571	24.066	1.014
opt EWMA	(0.16,2.66)		199.231	80.659	39.079	22.504	14.909	10.702	8.314	6.708	5.637	4.839	4.268	3.776	23.735	1.000
con CUSUM (k,h)	(0.5,10.35)		182.532	68.417	35.180	21.852	15.570	11.847	9.524	8.002	6.832	6.023	5.376	4.880	26.430	1.114
opt CUSUM	(0.88,5.76)		216.365	91.618	43.563	24.284	15.423	10.969	8.262	6.511	5.339	4.550	3.961	3.502	23.877	1.006
con EWMA (λ, L)	(0.2,2.58)	0.75	278.776	146.208	80.579	47.335	30.185	20.891	14.930	11.498	8.769	7.100	5.922	5.026	40.724	1.043
opt EWMA	(0.15,2.5)		256.512	133.970	72.278	42.933	28.223	19.882	14.721	11.307	9.004	7.393	6.294	5.465	39.686	1.016
con CUSUM (k,h)	(0.5,18.36)		234.056	109.840	60.862	39.359	27.921	21.183	17.080	14.089	12.065	10.429	9.258	8.311	45.775	1.172
opt CUSUM	(1.09,7.27)		264.368	143.623	77.411	45.373	28.632	19.957	14.443	10.862	8.473	6.724	5.702	4.893	39.060	1.000
con EWMA (λ, L)	(0.2,2.36)	0.9	302.668	203.706	131.149	83.078	56.026	38.601	27.352	19.781	14.466	11.148	8.330	6.740	66.684	1.075
opt EWMA	(1,2.7)		311.969	218.871	142.723	93.519	60.926	42.095	28.355	19.401	12.221	7.428	4.184	2.372	62.748	1.011
con CUSUM (k,h)	(0.5,32.87)		289.531	175.673	108.110	72.287	53.013	39.857	31.601	25.810	21.686	18.659	16.352	14.616	81.957	1.321
opt CUSUM	(2.37,0.43)		312.134	216.540	141.778	92.661	61.081	40.727	27.530	18.683	12.345	7.492	4.216	2.458	62.036	1.000

It is clear from Table 4.6 that when the process is uncorrelated ($\phi_1 = 0$) or positively correlated with ϕ_1 = 0.25 and 0.90 the conventional EWMA and CUSUM modified charts are very sensitive to small shifts (usually $\delta \leq 1.25$) and conversely the optimized EWMA and CUSUM modified charts perform relatively better for medium to large shifts ($\delta \geq 1.5$). Furthermore with $\phi_1 = 0.5$ and 0.75 the conventional CUSUM and optimized EWMA modified charts perform relatively better for small shifts ($\delta \leq 1.25$) and the conventional EWMA and optimized CUSUM modified charts also perform better for shifts ($\delta \geq 1.5$) than the other charts.

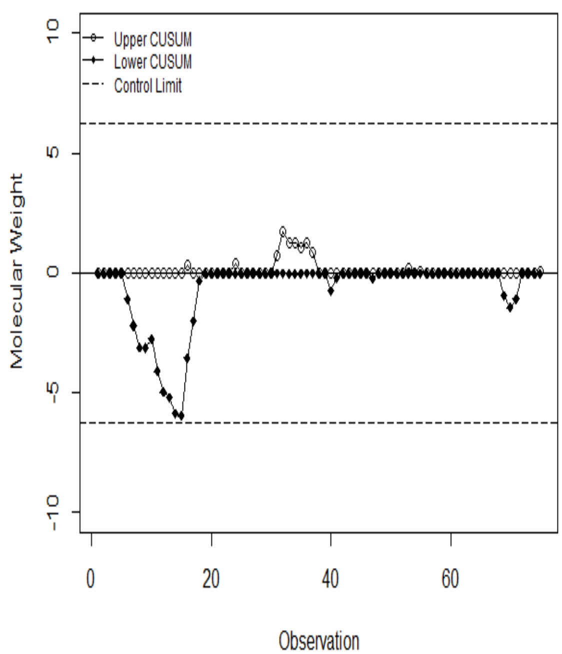
Consistently, it is noted that when $\phi_1 \leq 0.5$ the optimized EWMA modified chart followed by the either conventional EWMA or optimized CUSUM modified chart have the smallest *EQL* and *PCI* over the entire range. Additionally, with $\phi_1 \geq 0.75$ the optimized CUSUM followed by the optimized EWMA modified chart also have the smallest *EQL* and *PCI* values. Therefore, the conclusion is that the optimized EWMA and CUSUM modified charts have the best performance ability and relative efficiency to the others for $\phi_1 \leq 0.5$ and $\phi_1 \geq 0.75$ respectively.

4.4. Illustrative Examples

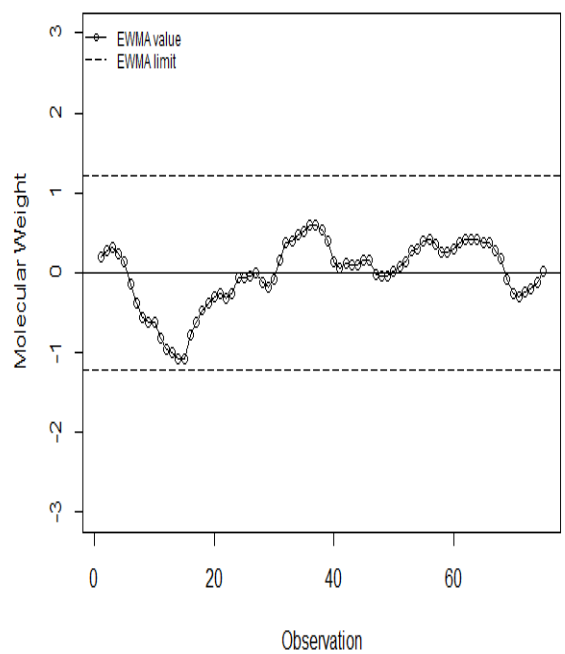
In these examples, we demonstrate how the optimized CUSUM and EWMA modified charts operate by using 2 sets of real data.

(a) Example 1

In this example, we use autocorrelated dataset from Montgomery (2009, pp. 491, Table 10E.9). This dataset consists of 75 molecular weight measurements from a process for which an AR (1) model with $\phi_1 = 0.67$ is a good fit. With the mean shift unknown, we choose optimal parameters for the charts to minimize *EQL* whilst keeping $ARL_0 = 370$. Hence (1.03, 6.23) and (0.12, 2.52) parameters are used in the CUSUM and EWMA modified charts respectively. The observations are standardized and the outputs from the charts are displayed in Figure 4.3. It is clear that the process is in-control in both charts with no point beyond the control limits.



(a) CUSUM chart

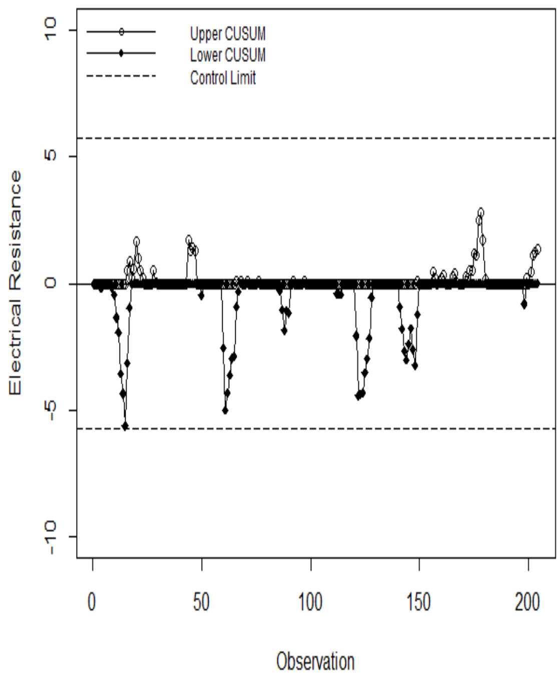


(b) EWMA chart

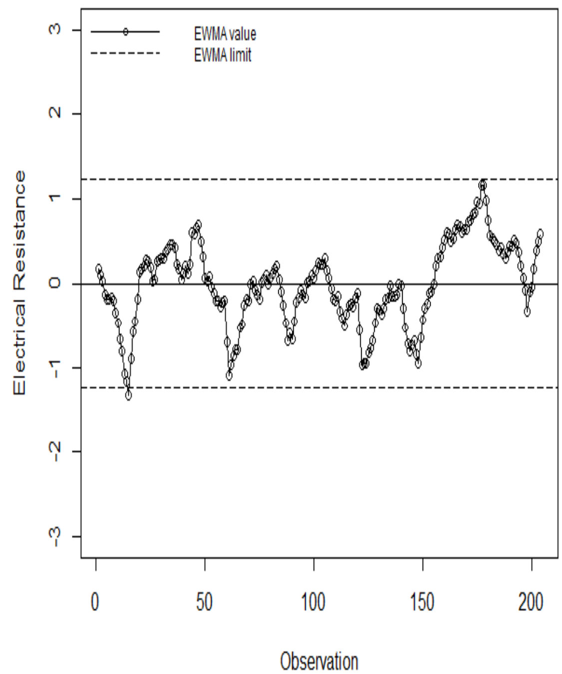
Figure 4.3: Modified charts for example 1

(b) Example 2

We use dataset from Shewhart (1931) which is made up of 204 electrical resistance measurements. AR (1) model with $\phi_1 = 0.55$ is a perfect fit. To obtain $ARL_0 = 370$, (0.95, 5.73) and (0.15, 2.63) parameters are used in the CUSUM and EWMA modified charts respectively to optimize the performance of the charts with respect to the *EQL*. The outputs from the charts are displayed in Figure 4.4 using the standardized observations. It is clear that observation 15 is out-of-control in the EWMA modified chart and we also suspect observation 15 is out-of-control in the CUSUM modified chart.



(a) CUSUM chart



(b) EWMA chart

Figure 4.4: Modified charts for example 2

4.5. Conclusions

Autocorrelation is a regular occurrence in many processes. The use of the modified chart helps in dealing with the effects due to serial correlation. In this work, we reviewed the performance and design of the CUSUM and EWMA modified charts. An exhaustive search optimization algorithm is used to determine parameters that optimize the performance of the charts using some control chart performance measures such as *ARL*, *EQL* and *PCI*.

In process monitoring when the magnitude of the shift is fixed or known, parameters for the charts can be selected in order to optimize the performance of the chart with respect to out-of-control *ARL* value. In reality, this magnitude of the mean shift is often unknown and any choice of the design parameters may lead to the poor performance of the charts. In such circumstances, the use of optimal parameters that minimize the *EQL* criterion is ideal, because the loss due to poor quality is reduced by the choice of these optimal parameters. The work provides the practitioner with optimal parameters to use in the charts for some selected autocorrelation levels.

CHAPTER 5

BIVARIATE CONTROL CHARTS FOR NON-NORMAL PROCESSES BASED ON DISPERSION ESTIMATES

5.1. Introduction

Variation is an unavoidable alteration in the conditions of a system or a process. The occurrence of variation can be monitored and detected using Statistical Process Control (SPC) tools such as control charts. When an out-of-control signal is detected, one should investigate the cause of the alarm and find a corrective measure for the variation if it is unnatural. Control charts can be classified into two groups depending on the number of quality characteristics that are to be monitored. Univariate control charts monitor one quality characteristic whilst multivariate charts monitor two or more quality characteristics. The use of univariate control charts to monitor quality characteristics separately ignores the correlation between the variables and this action can lead to invalid conclusions about the process (Jackson (1959), Alt and Smith (1988), Montgomery (2009)). In reality it is observed that most processes possess more than one quality characteristic which are correlated with each other (Montgomery (2009)). For example, in a wrought rod production company, the standard quality of the rods produced depends on the tensile strength, yield strength and modulus elasticity properties which are correlated. Multivariate charts are used to simultaneously monitor the quality variables of the process by using a single chart. The charts can be constructed to monitor variation in the process as a result of shifts in the mean vector, covariance matrix or both which can be attributed to special causes.

Multivariate control charts just like the univariate control charts are based on the assumption that observations from the process are normally distributed (Montgomery (2009)). The charts may malfunction in the event of a violation of this assumption. Factors such as extreme values (outliers), measurement error from devices and insufficient quantity of data collected can lead to a violation of the assumption. Furthermore, some processes in nature produce observations which follow non-normal distributions such as log-normal, exponential, Weibull, *t* and Gamma. James (1989), Bissell (1994) and Levinson and Polny (1999) observed that quality characteristics such as capacitance, insulation resistance and mold dimension follow non-normal distributions by nature. Recently a lot of work has been done in the use of univariate control charts to monitor process variability with various dispersion charts, based on normal and non-normal parent distributions (Stoumbos and Reynolds (2000), Riaz and Saghir (2007), Abbasi and Miller (2011), Abbasi *et al*. (2014), Saghir and Lin (2014)).

Works by authors such as Alt (1985), Alt and Smith (1988), Aparisi *et al*. (2001), Surtihadi *et al*. (2004), Yeh *et al*. (2006), Vargas and Lagos (2007), Costa and Machado (2008,2009) and the references therein have discussed extensively about multivariate charts for monitoring process variability based on multivariate normal distributions. Aparisi *et al*. (2001) presented the $|S|$ chart with adaptive sample size to monitor process variability and they concluded that the power of the proposed scheme was significantly improved in most cases as compared to the $|S|$ chart with fixed sample size. Yeh *et al*. (2003) designed a multivariate exponentially weighted moving average chart (EWMA V-chart) using the concept of probability integral transformation. The chart is observed to be sensitive in detecting small shifts in process variability than the $|S|$ chart for normal processes. Khoo and Quah (2004) presented a variety of multivariate charts which were formulated by transforming the standard chart based on the $|S|$ quantity for monitoring multivariate dispersion into a standard scale such that run rules were incorporated. Costa and Machado (2009, 2011) proposed the *VMAX* and *RMAX* charts, which are based on the sample standardized variances and ranges statistics respectively for multivariate process monitoring.

With multivariate observations departing form normality, not much work has been done on the subject using dispersion charts. Riaz and Does (2008) proposed a bivariate dispersion chart which is centred on the generalized Gini mean differences $|G|$. The $|G|$ chart was observed to be more robust to departures from normality than the $|S|$ chart in the work. Also, they remarked that the $|S|$ chart loses its effectiveness in situations where the observations are obtained from contaminated multivariate normal distributions or violate the normality assumption. Another drawback of using the $|S|$ chart is that different matrices can yield the same $|S|$ value and therefore process dispersion may go off track but remain undetected (Alt (1973)). Similarly, Saghir (2015) also compared the performances of the $|S|$ and $|G|$ charts for monitoring bivariate processes using a variety of non-normal distributions. In this chapter to monitor process variability, we will concentrate on the use of efficient estimators of dispersion and further make their use for the monitoring of bivariate observations by considering bivariate non-normal distributions (*t* and Gamma). We present control charts based on the sample standardized dispersion estimates such as the standard deviation (*S*), interquartile range (*Q*), average absolute deviation from median (*MD*) and the median absolute deviation (*MAD*) for non-normal bivariate processes. The performance of the charts are assessed and compared with the *RMAX* and $|S|$ charts.

The rest of this chapter is planned as follows: Section 5.2 discusses some dispersion estimates. Section 5.3 examines bivariate dispersion control charts for detecting shifts in the covariance matrix of a process; the performance evaluation and comparison of the charts are discussed in Sections 5.4 and 5.5 respectively. Illustrative examples are presented to demonstrate how the charts operate in Section 5.6 and finally conclusions are summarised in Section 5.7.

5.2. Dispersion Estimates

In this section, we describe dispersion estimates which are used in formulating the control charts for process monitoring. Let *X* represents a quality characteristic of a process and $X_1, X_2, X_3, ..., X_n$ be a random sample of size *n* generated from a distribution with mean and standard deviation as μ and σ respectively. Also, let $X_{(i)}$ be the *i*th order statistic (smallest to largest) and denote the sample mean, sample median and absolute value of X by $\bar{X}, \tilde{X}$ and $|X|$ respectively. Based on these notations, we define the following dispersion estimates as;

Sample range

This statistic is defined as the difference between the maximum and minimum observations, i.e.

$$R = X_{(n)} - X_{(1)} \tag{5.1}$$

With a small sample size, the R statistic is observed to be an efficient estimate of dispersion but as sample size increases, it becomes less efficient (Abbasi (2012)). It is well known that the range statistic is a very sensitive estimator of dispersion and can be highly affected in the presence of outliers or non-normality. Hence, the performance of range based chart is more likely to perform very badly in the absence of normality.

Sample standard deviation

The sample standard deviation (S) is represented as;

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2} \tag{5.2}$$

Similar to the R statistics, the S statistics is also sensitive to the existence of non-normality in the process observations but it is very effective for monitoring dispersion of normally distributed observations.

Interquartile range

The interquartile range (Q) statistic is the difference between the third quartile (Q_3) and first quartile (Q_1) observations, i.e.

$$Q = Q_3 - Q_1 \tag{5.3}$$

However, we use $Q = \frac{Q_3 - Q_1}{1.34898}$ in this work. This formulation of Q is used because it is often used in statistical software packages like Minitab and SPSS (Abbasi *et al*. (2014)). Furthermore, studies by authors such as Riaz (2008), Abbasi and Miller (2011) and Abbasi *et al*. (2014) used this representation of Q for dispersion monitoring.

Other dispersion estimators, such as the average absolute deviation from median (*MD*) and the median absolute deviation (*MAD*) can also be used. These estimators are centred on deviations from the sample median.

Absolute deviation from median

The absolute deviation from median (*MD*) statistics is represented as;

$$MD = \frac{1}{n}\sum_{i=1}^{n}\left|X_i - \tilde{X}\right| \tag{5.4}$$

The *MD* statistics is observed to be an effective estimator for monitoring observations which are non-normally distributed i.e. *t* and Gamma (Abbasi and Miller (2013)).

Median absolute deviation

The median absolute deviation (*MAD*) statistics is defined as;

$$MAD = 1.4826\, med\, \left|X_i - \tilde{X}\right| \tag{5.5}$$

These statistics are noted for their efficiency and robustness in detecting shifts in the dispersion parameter. We develop the bivariate control charts using these dispersion statistics.

The observations in the bivariate situation are represented as X_{ij} which is a $2\times n$ matrix of quality measurements, $i=1,2$ quality variables, $j=1,2,........,n$ number of observations in the sample.

5.3. Bivariate Control Charts for Dispersion

In this section, the dispersion control chart structures for monitoring shifts in the covariance matrix of bivariate processes are briefly discussed. Let *X* and *Y* represent two quality characteristics of a process which are correlated and follow a bivariate distribution with mean vector and covariance matrix given as $\underline{\mu}=\begin{pmatrix}\mu_x\\ \mu_y\end{pmatrix}$ and $\Sigma=\begin{bmatrix}\sigma_x^2 & \sigma_{xy}\\ \sigma_{xy} & \sigma_y^2\end{bmatrix}$ respectively.

We present bivariate dispersion charts which are constructed based on the standardized sample standard deviation (*S*), interquartile range (*Q*), average absolute deviation from median (*MD*) and median absolute deviation (*MAD*) statistics. These control charts are devised following the *VMAX* and *RMAX* charts of Costa and Machado (2009, 2011). The newly proposed charts together with some existing dispersion charts (i.e. *RMAX* and $|S|$) for processes that follow bivariate normal and non-normal distributions are discussed below;

5.3.1 Existing Bivariate Dispersion Charts

In this section, we examine the design structures for the *RMAX* and $|S|$ charts for monitoring shifts in the covariance matrix of bivariate processes.

Generalized variance $|S|$ chart

Alt (1985) proposed the $|S|$ chart which is based on the generalized variance $|S|$ statistics. It is observed that when a process with two quality characteristics is in-control, the expression $\frac{2(n-1)|S|^{1/2}}{|\Sigma_0|^{1/2}}$ is distributed as a chi-squared random variable having (*2n-4*) degrees of freedom (*v*), where $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$. S is the covariance matrix, S_{11} and S_{22} represent the sample variances of the variables and $S_{21} = S_{12}$ represent the sample covariances between the variables. For a specified Type I risk value (α), the *UCL* can be expressed as;

$$UCL = \frac{|\Sigma_0|(\chi^2_{2n-4,\alpha})^2}{4(n-1)^2} \tag{5.6}$$

The chart is considered as out-of-control when $|S|$ exceeds the *UCL*. For the non-normal parent distributions, *UCL* is selected in order to yield the desired in-control *ARL* value.

RMAX chart

Costa and Machado (2011) proposed the *RMAX* chart as an alternative chart to the $|S|$ chart for monitoring shifts in the covariance matrix of a process. This chart is based on the sample range. The *RMAX* statistics is represented as the maximum sample range of the characteristic variables in the bivariate processes. That is;

$$RMAX = \max\{R_1, R_2\} \tag{5.7}$$

Where R_i, is the range (*R*) of the *i*th quality characteristic for $i = 1, 2$. The *RMAX* chart is said to be out-of-control when the *RMAX* statistic exceeds the upper control limit (*UCL*).

5.3.2. Proposed Bivariate Dispersion Charts

We present the design structures for the proposed bivariate dispersion charts (i.e. *SMAX*, *QMAX*, *MDMAX* and *MADMAX*) in this section. These charts are based on the maximum value of the different sample statistics for the bivariate process. These charts are represented as;

$$SMAX = \max\{S_1, S_2\} \tag{5.8}$$

$$QMAX = \max\{Q_1, Q_2\} \tag{5.9}$$

$$MDMAX = \max\{MD_1, MD_2\} \tag{5.10}$$

$$MADMAX = \max\{MAD_1, MAD_2\} \tag{5.11}$$

Where S_i, Q_i, MD_i and MAD_i are the *S, Q, MD* and *MAD* statistics respectively of the *i*th quality characteristic, for $i = 1, 2$. An out-of-control signal is generated when the *SMAX*, *QMAX*, *MDMAX* and *MADMAX* statistics exceed their respective *UCL*. *UCL* can be selected in order to yield the desired in-control average run length (*ARL*) value.

5.4. Performance Evaluation

The performance assessment of the charts is conducted using the average run length (*ARL*) measure. The control charts are said to be in a state of statistical control when

$$\Sigma_0 = \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{bmatrix} \tag{5.12}$$

An out-of-control situation arises when Σ_0 changes to Σ_1 or Σ_2 due to special causes that may affect the process. In this paper, we define the out-of-control conditions as;

$$\Sigma_1 = \begin{bmatrix} a_1.a_1.\sigma_{11} & a_1.\sigma_{12} \\ a_1.\sigma_{21} & \sigma_{22} \end{bmatrix} \qquad \Sigma_2 = \begin{bmatrix} a_1.a_1.\sigma_{11} & a_1.a_2.\sigma_{12} \\ a_2.a_1.\sigma_{21} & a_2.a_2.\sigma_{22} \end{bmatrix} \tag{5.13}$$

Case 1: The shift affects only the first quality characteristic represented by Σ_1.

Case 2: The shift affects both quality characteristics represented by Σ_2.

Here, σ_{ij} represents the variances for $i = j = 1,2$ and the covariances between the quality characteristics for $i \neq j = 1,2$. $a_i \geq 1, i = j = 1,2$ is the value of the shift and $\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}$, $i \neq j = 1,2$ is the correlation coefficient.

We investigate the performance of the charts for bivariate processes which are normally and non-normally distributed. The bivariate *t* (*v*) and Gamma (α, β) distributions will be considered to represent heavy-tailed symmetric and skewed distributions respectively. The parameters *v*, α and β represent the degrees of freedom, shape and the rate of the respective distributions. *ARL* values are presented for the charts, for $n = \{5, 10\}$ and $\rho = \{0, 0.3, 0.7\}$. We consider bivariate *t*(3), *t*(5), *t*(10), *t*(50), Gamma(2,1) ,Gamma (2,2), Gamma (3,1), Gamma (10,1) and Gamma(1,1) (bivariate exponential) distributions. Shifts of magnitude $a_i \geq 1$, $i = j = 1, 2$ are induced into the covariance matrix to yield Σ_1 or Σ_2. The performances of the proposed charts herein are investigated for increases in the covariance of the process. 10,000 simulations are performed to obtain estimates of the *ARL* values. The *UCL* is adjusted in each chart to yield the anticipated in-control *ARL* value of 200 for each correlation parameter considered for the normal and non-normal processes. The simulation procedures were computerized using R software.

5.5. Comparison of the Charts

Multiple quality variables in a steel rod production company that can be monitored may include the length and thickness of the rods which are correlated with each other. The shift in the process can occur only in the length or thickness of the rods or even both. Therefore a comparison of the charts is conducted for assignable causes that affect one or both quality characteristics of the process. These charts are assessed using the *ARL* criterion for the multivariate normal and non-normal processes. The *ARL* values together with the design parameters for the respective charts are displayed in Tables 5.1 to 5.3 and Appendix Tables A1 to A3 for the bivariate normal, *t* and Gamma distributed processes using *n*=5 and 10. Figures 5.1 and 5.2 display the graphs of the *ARL* results for shifts in only one quality variable whilst Figures 5.3 and 5.4 are graphs for shifts in both quality variables of the process. The discussions on the performance of the charts are grouped under the bivariate normal and non-normal distributed processes.

Table 5.1: *ARL* values for the bivariate charts when *n*=5 and $ARL_0 = 200$ for the bivariate normal process

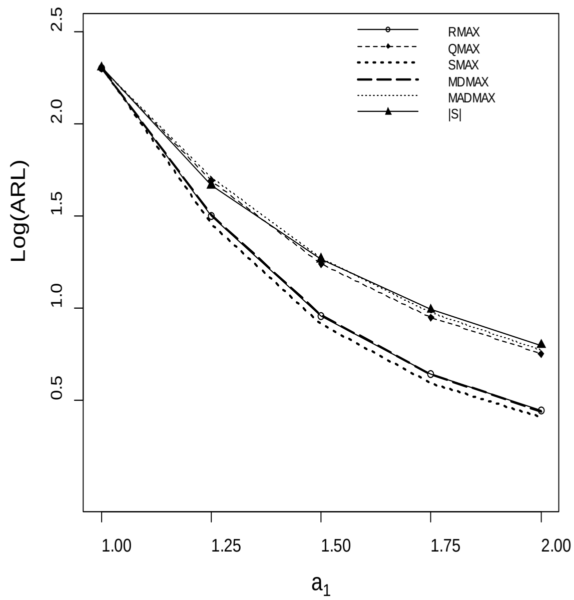
		RMAX			*QMAX*			*SMAX*			*MDMAX*			*MADMAX*			|S|		
	ρ	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
	UCL	5.154	5.15	5.128	2.241	2.24	2.23	2.029	2.024	2.017	1.477	1.476	1.471	2.57	2.5657	2.56	5.375	4.8914	2.7413
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	202.15	198.21	200.94	200.95	199.92	203.52	203.41	202.34	200.07	201.09	200.63	199.42	201.80	203.58	201.39	202.82	201.47	200.90
1.25	1	31.91	31.71	30.80	49.45	48.84	48.15	28.61	28.43	28.37	32.20	31.95	31.69	51.73	50.78	50.71	46.76	46.92	46.65
1.5	1	9.14	9.10	8.99	17.43	17.35	17.28	8.29	8.11	8.02	9.21	9.18	9.16	18.80	18.51	18.32	18.48	18.62	18.23
1.75	1	4.40	4.39	4.30	8.94	8.83	8.81	3.95	3.90	3.88	4.38	4.43	4.46	9.48	9.40	9.31	9.85	9.70	9.82
2	1	2.79	2.76	2.75	5.66	5.65	5.63	2.57	2.56	2.52	2.77	2.82	2.88	5.98	5.95	5.95	6.32	6.28	6.25
1.25	1.25	17.37	17.40	18.57	27.77	28.18	28.74	15.78	15.79	16.78	17.22	17.48	18.71	29.14	29.92	29.84	15.47	15.32	15.23
1.25	1.5	7.38	7.63	7.79	13.80	14.15	14.43	6.68	6.80	7.14	7.50	7.70	7.94	14.96	15.18	15.34	7.81	7.79	7.79
1.5	1.5	4.87	4.96	5.40	9.35	9.40	10.01	4.41	4.52	4.98	4.99	5.03	5.61	10.00	10.03	10.33	4.51	4.57	4.48
1.5	1.75	3.22	3.26	3.52	6.27	6.39	6.64	2.93	2.94	3.30	3.29	3.35	3.66	6.69	6.71	6.86	3.18	3.17	3.22
1.75	2	1.96	2.02	2.20	3.73	3.79	4.03	1.84	1.88	1.99	1.98	2.02	2.24	3.95	3.98	4.14	1.97	1.98	1.98
2	2	1.70	1.72	1.89	3.11	3.14	3.37	1.59	1.63	1.74	1.73	1.74	1.94	3.32	3.38	3.50	1.68	1.70	1.69

Table 5.2: *ARL* values for the bivariate charts when *n*=5 and $ARL_0 = 200$ for the bivariate *t*(5) process

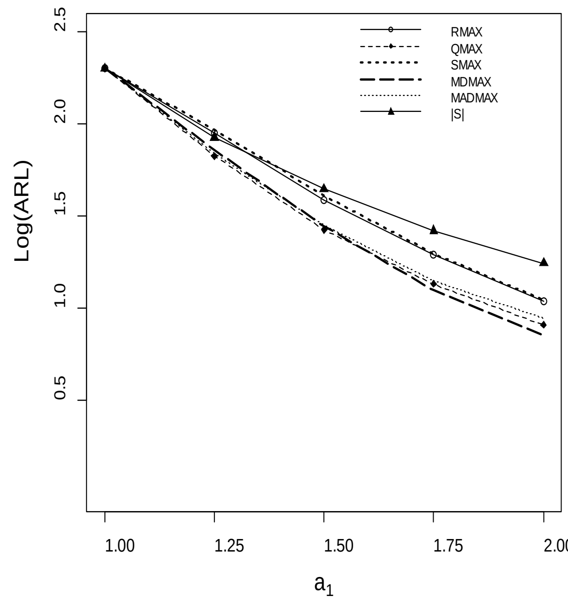
		RMAX			*QMAX*			*SMAX*			*MDMAX*			*MADMAX*			|S|		
	ρ	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
	UCL	7.411	7.402	7.246	2.214	2.21	2.1969	2.946	2.929	2.873	1.823	1.8207	1.788	2.6	2.599	2.589	10.06	9.11	5.099
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*		*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	202.44	202.52	200.07	201.13	202.18	201.96	200.55	199.82	201.03	201.09	200.97	199.76	200.60	199.39	201.84	200.21	199.78	200.27
1.25	1	89.57	88.47	87.68	67.04	66.85	66.76	93.41	92.06	90.05	72.72	71.79	70.43	68.92	68.30	68.05	84.76	85.23	85.68
1.5	1	38.51	38.23	36.81	26.73	26.23	26.06	41.20	39.28	37.42	27.77	26.89	25.23	28.33	28.05	27.61	44.38	44.36	43.10
1.75	1	19.58	19.30	17.95	13.62	13.60	13.07	19.93	19.12	18.06	12.65	12.47	11.92	14.13	14.41	14.81	26.47	26.25	26.55
2	1	10.89	10.78	10.22	8.16	8.15	8.09	11.11	10.92	10.10	7.15	6.90	6.68	8.79	8.57	8.52	17.58	17.24	17.53
1.25	1.25	58.29	58.91	59.14	40.27	40.67	41.11	60.41	61.46	62.65	44.90	45.67	45.99	41.71	42.43	42.88	37.85	38.60	37.59
1.25	1.5	31.35	32.27	32.33	21.14	21.19	21.77	33.69	33.76	34.07	22.39	22.77	22.88	21.47	22.36	22.91	21.59	21.03	21.04
1.5	1.5	22.34	22.73	23.20	14.57	14.90	15.39	23.45	23.77	24.66	15.55	15.91	15.98	15.49	15.66	15.94	12.32	12.29	12.50
1.5	1.75	14.44	14.71	14.76	9.57	9.70	10.28	15.01	15.18	15.44	9.60	9.63	9.69	10.30	10.34	10.59	8.18	8.36	8.36
1.75	2	7.70	7.99	8.12	5.53	5.69	5.89	7.96	8.02	8.15	5.07	5.17	5.41	5.80	5.85	6.27	4.45	4.42	4.39
2	2	6.09	6.31	6.50	4.53	4.54	4.82	6.22	6.32	6.55	4.11	4.19	4.32	4.77	4.88	5.08	3.47	3.46	3.41

Table 5.3: *ARL* values for the bivariate charts when n=5 and $ARL_0 = 200$ for the bivariate Gamma (1, 1) process

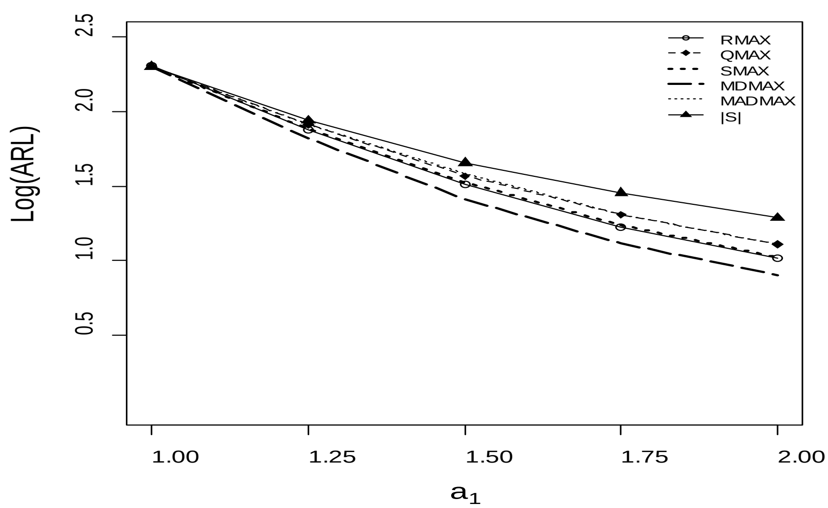
		RMAX			*QMAX*			*SMAX*			*MDMAX*			*MADMAX*			|S|		
	ρ	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
	UCL	7.397	7.34	7.2539	2.628	2.622	2.598	3.11	3.093	3.057	1.898	1.894	1.87	2.53	2.52	2.508	11.92	10.85	6.08
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	202.62	200.03	200.32	200.28	199.99	201.94	199.36	200.98	200.08	200.29	199.92	200.57	200.31	200.67	200.71	200.32	200.02	201.10
1.25	1	75.22	73.93	73.69	81.78	80.92	80.38	77.03	76.00	75.86	65.94	65.65	64.50	81.94	80.88	80.67	86.97	85.50	84.78
1.5	1	32.23	31.15	31.09	36.73	36.61	36.10	33.43	32.65	32.36	25.94	25.67	25.09	38.03	36.88	36.46	45.16	45.07	44.72
1.75	1	16.77	16.29	15.98	20.39	19.86	19.36	17.42	17.14	16.75	13.05	12.89	12.52	20.41	20.11	20.09	28.50	27.49	28.14
2	1	10.34	10.13	10.03	12.85	12.60	12.60	10.47	10.42	10.21	7.95	7.89	7.75	12.92	12.90	12.56	19.32	19.05	18.88
1.25	1.25	46.91	46.92	49.56	51.04	51.25	53.13	46.78	47.46	50.93	39.77	40.30	42.77	50.96	51.25	53.36	40.42	39.15	39.55
1.25	1.5	25.91	25.93	26.50	28.96	29.42	30.35	26.13	26.38	27.63	20.39	21.10	21.80	28.74	28.83	29.95	23.65	22.99	23.03
1.5	1.5	18.03	18.05	19.05	20.32	20.56	21.49	18.30	18.41	20.01	14.11	14.69	15.63	20.62	20.63	20.95	14.60	13.96	13.94
1.5	1.75	12.01	12.13	13.26	14.01	14.18	15.00	12.39	12.42	13.31	9.22	9.60	10.37	14.07	14.10	14.74	9.83	9.55	9.84
1.75	2	6.79	6.90	7.53	8.25	8.54	9.07	6.98	7.23	7.88	5.32	5.58	6.13	8.30	8.43	8.89	5.48	5.52	5.52
2	2	5.52	5.69	6.24	6.82	6.92	7.35	5.62	5.85	6.57	4.29	4.54	4.96	6.83	6.97	7.20	4.38	4.46	4.38



(a) Normal

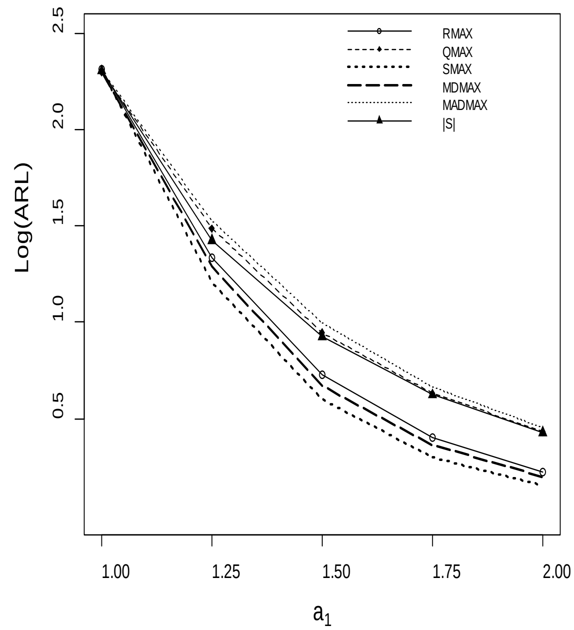


(b) t_5

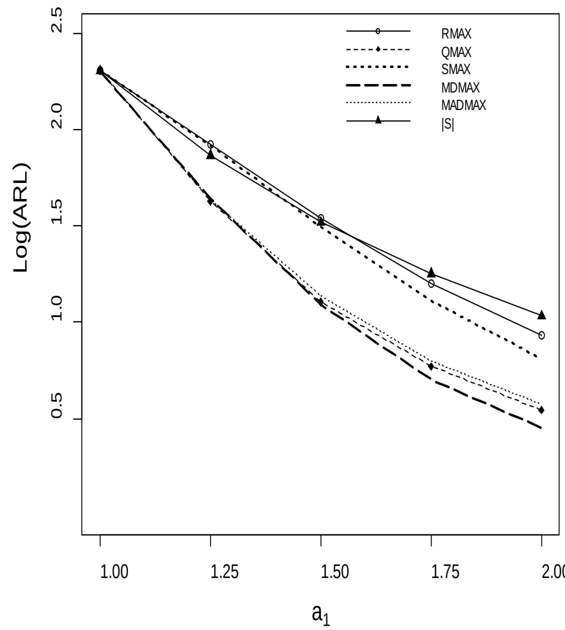


(c) Gamma (1, 1)

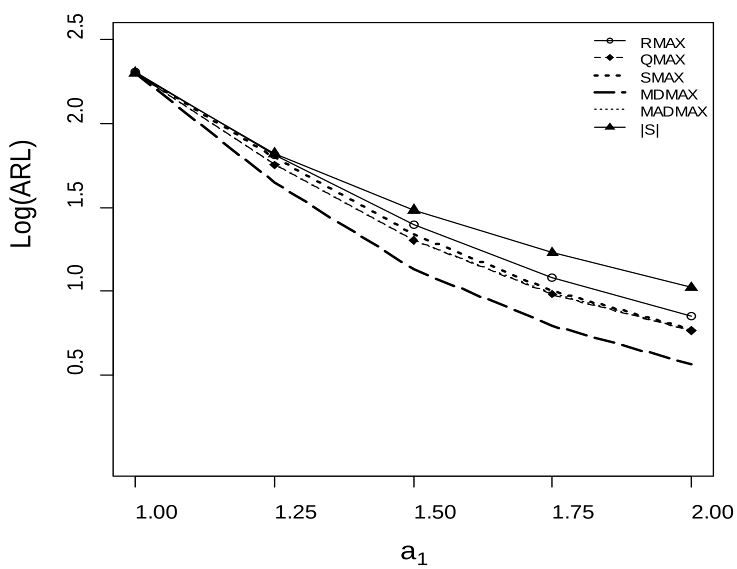
Figure 5.1: *ARL* curves for the different bivariate dispersion charts with shift in only one quality variable (i.e. $a_2 = 1$) when n=5, $\rho = 0$ and $ARL_0 = 200$



(a) Normal

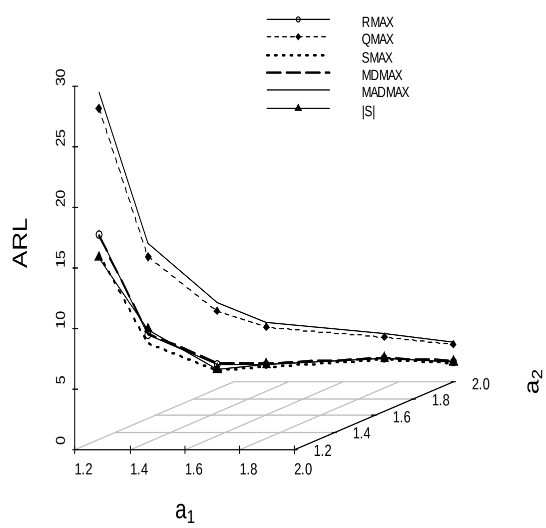


(b) t_5

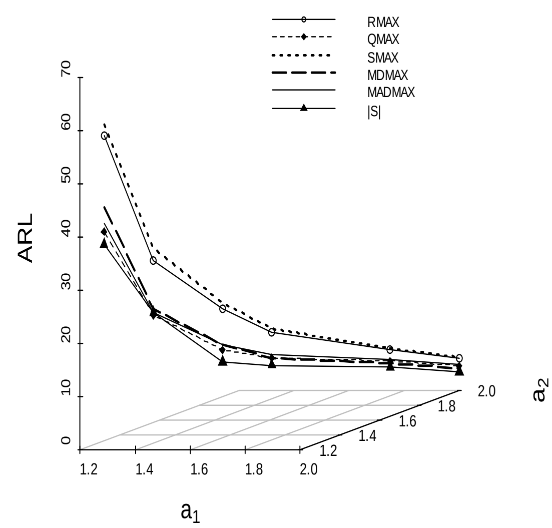


(c) Gamma (1, 1)

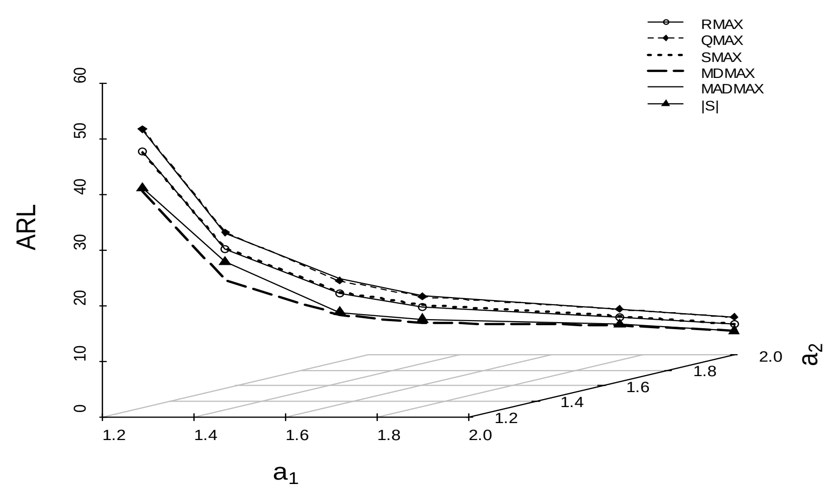
Figure 5.2: *ARL* curves for the different bivariate dispersion charts with shift in only one quality variable (i.e. $a_2 = 1$) when n=10, $\rho = 0$ and $ARL_0 = 200$



(a) Normal

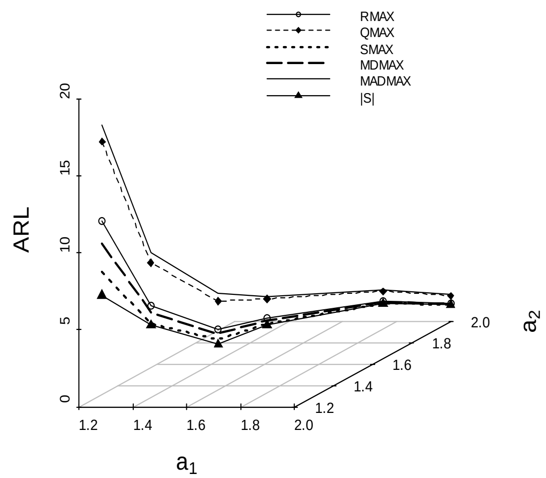


(b) t_5

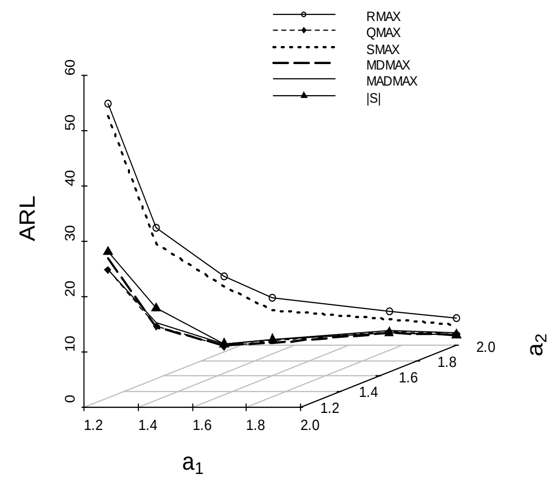


(c) Gamma (1, 1)

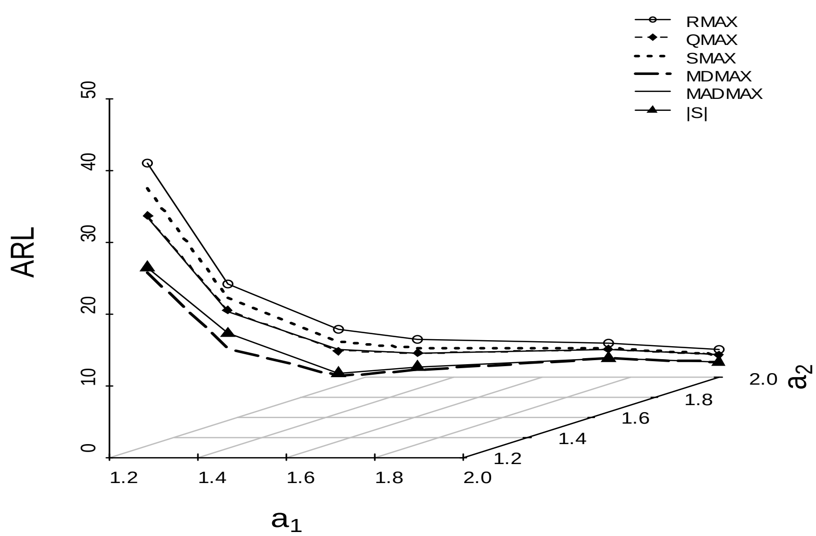
Figure 5.3: *ARL* curves for the different bivariate dispersion charts with shifts (a_1, a_2) in both quality variables when *n*=5, $\rho = 0$ and $ARL_0 = 200$



(a) Normal



(b) t_5



(c) Gamma (1, 1)

Figure 5.4: *ARL* curves for the different bivariate dispersion charts with shifts (a_1, a_2) in both quality variables when n=10, $\rho = 0$ and $ARL_0 = 200$

Bivariate Normal distribution

By Comparing the *ARL* values of the charts in Tables 5.1 and A1, the *SMAX* and $|S|$ charts slightly perform better than the *MDMAX* and *RMAX* charts for assignable causes that affect both quality variables of the process depending on *n*. The *QMAX* and *MADMAX* charts are the least performing charts for special causes that affect both quality variables because they had slightly higher ARL_1 values.

Furthermore, it is observed that the *SMAX* chart has the best performance ability for assignable causes that affect only one quality characteristic because it had the smallest ARL_1 values for *n* (i.e. 5 and 10). This performance of the *SMAX* chart is followed by the *MDMAX* and *RMAX* charts which perform almost the same especially for small *n* (e.g. 5). Note that for large values of *n*, the *MDMAX* chart performs slightly better than the *RMAX* chart. The *QMAX*, *MADMAX* and $|S|$ charts are inferior to the other charts because they have relatively higher ARL_1 values.

Bivariate *t* distribution

As discussed in Section 1.2.3., quality variables from some processes are non-normal by nature (may follow bivariate *t* distribution). From Tables 5.2 and A2, it is clear that the $|S|$ chart performs better than the other charts with respect to assignable causes that affect both quality variables for v= 5 when n=5. However, when n=10, the performance of the *MDMAX*, *QMAX*, *MADMAX* and $|S|$ charts are almost the same. The *RMAX* and *SMAX* charts are inferior to the other charts with respect to *ARL* performance when the assignable causes affect both quality characteristics.

The *MDMAX* chart performs better than the other charts for shifts $a_1 > 1.25$ in only one quality characteristic of the bivariate process. However, for small shifts $a_1 \leq 1.25$ in one quality characteristic, the *QMAX* and *MADMAX* charts perform slightly better than the *MDMAX* and the other charts. Generally, the *RMAX*, *SMAX* and $|S|$ charts are the least performing charts and are less sensitive to the special causes that affect only one quality characteristic.

Bivariate Gamma distribution

As discussed in Section 1.2.3., several quality characteristics in industrial operations (e.g. mining processes) follow skewed distributions such as the bivariate Gamma distribution by nature. In Tables 5.3 and A3, it is observed that for special causes that affect both quality characteristics of the bivariate Gamma distributed process, the *MDMAX* chart closely followed by the $|S|$ chart relatively outperform the other charts. However, when *n*=5 the *RMAX* and *SMAX* charts perform slightly better than the *QMAX* and *MADMAX* charts. Further, when *n*=10 the *QMAX*, *MADMAX* and *SMAX* charts perform almost the same for assignable causes that affect both quality characteristics for bivariate Gamma processes.

The *MDMAX* chart performs better than the other charts for shifts in one quality characteristic. When *n*=5 the *RMAX* and *SMAX* charts perform almost the same and slightly better than the *QMAX* and *MADMAX* charts. Furthermore, when *n*=10 the *QMAX*, *MADMAX* and *SMAX* charts perform almost the same. Note that the $|S|$ chart is less sensitive to shifts that affect only one quality variable as compared to the other charts.

Table 5.4: Effect of v on ARL values for the different bivariate dispersion charts when n=5, $\rho = 0$ and $ARL_0 = 200$

t (3)

		RMAX	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	|*S*|
	UCL	10.1861	2.072	4.165	2.286	2.367	14.388
a1	a2	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	200.16	201.37	200.97	200.84	201.48	200.41
1.25	1	128.84	90.10	131.25	117.18	78.19	112.34
1.5	1	80.26	38.99	84.87	63.30	36.55	67.37
1.75	1	51.20	19.85	53.24	36.89	18.45	49.40
2	1	31.67	12.29	35.17	22.38	11.67	35.60
1.25	1.25	96.57	56.67	104.93	82.13	53.57	63.59
1.25	1.5	65.99	30.23	71.18	58.98	28.70	38.65
1.5	1.5	52.58	21.39	55.85	42.17	20.29	27.27
1.5	1.75	37.36	14.49	41.08	28.49	13.09	17.98
1.75	2	24.24	8.11	26.73	16.54	7.81	9.83
2	2	20.04	6.99	22.49	13.08	6.43	7.83

t (10)

		RMAX	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	|*S*|
	UCL	5.986	2.236	2.34	1.597	2.6149	7.07
a1	a2	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	200.63	198.40	203.94	200.53	199.54	201.04
1.25	1	53.71	56.58	51.93	44.57	58.79	62.28
1.5	1	18.06	21.35	16.23	14.01	22.71	27.26
1.75	1	8.10	10.52	7.35	6.61	11.49	15.30
2	1	4.70	6.58	4.28	3.87	7.10	9.96
1.25	1.25	30.85	33.26	29.35	25.33	35.28	23.42
1.25	1.5	14.44	16.51	13.41	11.47	17.71	11.46
1.5	1.5	9.67	11.18	8.72	7.48	12.33	6.89
1.5	1.75	5.89	7.47	5.35	4.78	8.03	4.63
1.75	2	3.22	4.30	2.98	2.67	4.72	2.58
2	2	2.67	3.62	2.43	2.23	3.87	2.15

t (50)

		RMAX	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	|*S*|
	UCL	5.276	2.2374	2.073	1.4944	2.578	5.655
a1	a2	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	197.10	198.83	200.23	201.16	199.78	200.86
1.25	1	34.33	50.28	31.68	33.01	53.07	50.37
1.5	1	9.94	17.85	9.10	9.63	19.16	19.85
1.75	1	4.72	9.21	4.33	4.62	9.71	10.80
2	1	2.99	5.70	2.74	2.93	6.15	6.83
1.25	1.25	19.31	28.22	17.25	18.75	30.75	16.81
1.25	1.5	8.35	14.11	7.57	8.08	15.10	8.43
1.5	1.5	5.45	9.52	4.94	5.29	10.11	4.90
1.5	1.75	3.54	6.41	3.18	3.45	6.84	3.39
1.75	2	2.11	3.81	1.96	2.07	4.09	2.07
2	2	1.80	3.15	1.67	1.78	3.37	1.76

Effect of ρ and *v* on the charts

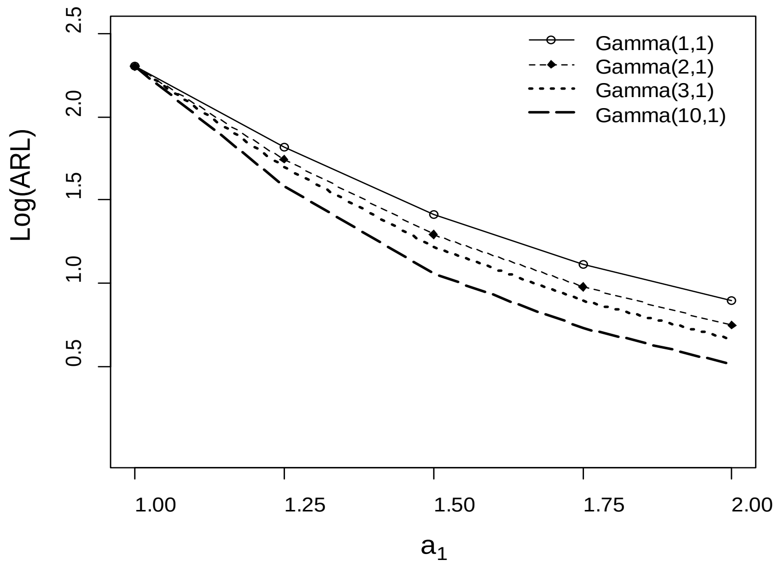
It is noticed from Tables 5.1 to 5.3 and A1 to A3 that the influence of ρ on the ARL_1 values is not much significant in the charts for the bivariate normal, *t* and Gamma processes. It is observed that as ρ increases, the ARL_1 values decrease slightly in the *RMAX*, *SMAX*, *MDMAX* and *MADMAX* charts when the shift occurs in one quality characteristic. But when the shift occurs in both quality characteristics, the ARL_1 values increase slightly in the *RMAX*, *SMAX*, *MDMAX* and *MADMAX* charts as ρ increases. These findings concerning ρ in the *SMAX*, *QMAX*, *MDMAX* and *MADMAX* charts are consistent with that in the *RMAX* chart of Costa and Machado (2011) for the bivariate normal process.

Table 5.4 displays the *ARL* values of the charts with different *v* for the bivariate *t* distributed process. It is noted that with a small *v* (i.e. 3), the *MADMAX* chart is immediately trailed by the *QMAX* chart and it performs better than the other charts with respect to shifts in one or both quality characteristic. Note that with *v* =10, the *MDMAX* chart followed by the *SMAX* chart outperforms the other charts for assignable causes that affect one quality characteristic and the $|S|$ chart also followed by the *MDMAX* chart performs better for shifts in both quality characteristics of the process. Furthermore, when *v*=50 the *SMAX* chart followed by the *MDMAX* and *RMAX* charts performs better for shifts due to special causes that affect one quality characteristic whilst the *SMAX*, *RMAX*, *MDMAX* and $|S|$ charts slightly perform better than the other charts for assignable causes that affect both quality characteristics. As expected it is observed that as *v* becomes large, the bivariate *t* distribution approaches the bivariate normal distribution. Hence, the *ARL* values for the bivariate *t* distributed process become similar to that of the bivariate normally distributed process as *v* becomes large.

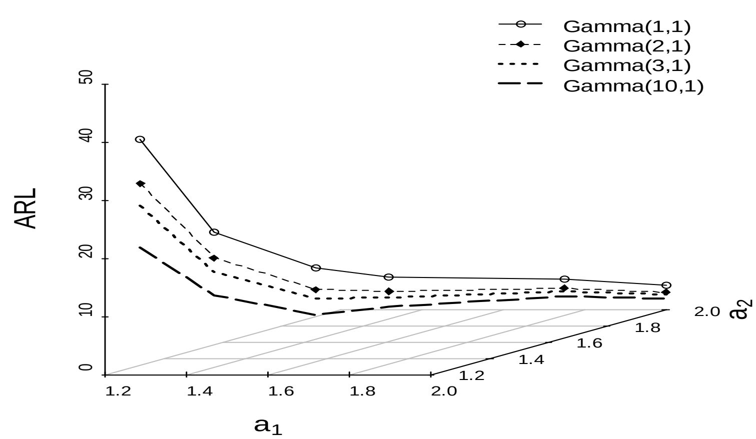
Table 5.5: Effect of α and β on *ARL* values for the different bivariate dispersion charts when *n*=5, $\rho = 0$ and $ARL_0 = 200$

		Gamma(2,1)						Gamma(2,2)					
		RMAX	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	*|S|*	*RMAX*	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	*|S|*
	UCL	6.581	2.484	2.72	1.745	2.54	8.982	6.59	2.484	2.72	1.74	2.536	8.98
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	198.88	200.45	201.53	199.18	204.25	198.62	201.59	200.04	201.24	199.61	200.26	199.92
1.25	1	63.93	71.17	64.59	55.73	69.33	71.96	65.23	70.50	65.99	55.49	68.04	73.05
1.5	1	24.91	29.17	25.36	19.69	27.86	35.01	24.50	29.13	25.50	19.64	28.38	35.42
1.75	1	11.75	14.99	12.22	9.54	14.80	21.08	11.98	14.99	12.14	9.40	14.81	21.08
2	1	7.00	9.24	7.07	5.64	8.91	13.64	7.12	9.42	7.12	5.57	8.87	13.93
1.25	1.25	38.02	42.29	39.79	32.17	41.78	31.00	38.15	42.22	39.67	32.48	41.61	30.89
1.25	1.5	19.46	23.47	19.76	15.91	22.08	16.72	19.36	22.54	19.82	15.80	21.64	16.78
1.5	1.5	13.19	15.88	13.33	10.43	15.23	9.88	13.10	15.57	13.31	10.47	15.44	9.78
1.5	1.75	8.70	10.59	8.80	6.68	10.41	6.75	8.57	10.58	9.01	6.70	10.30	6.61
1.75	2	4.71	6.11	4.87	3.78	5.91	3.77	4.72	6.20	4.85	3.78	5.86	3.77
2	2	3.79	5.01	3.95	3.08	4.90	3.01	3.93	5.09	3.93	3.11	4.76	2.95

		Gamma (3,1)						Gamma(10,1)					
		RMAX	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	*|S|*	*RMAX*	*QMAX*	*SMAX*	*MDMAX*	*MADMAX*	*|S|*
	UCL	6.238	2.424	2.548	1.672	2.532	8.02	5.562	2.306	2.232	1.546	2.548	6.26
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	200.86	201.20	199.35	199.95	199.02	202.45	200.89	200.66	201.62	199.76	200.62	199.77
1.25	1	60.03	64.80	56.79	50.19	62.83	69.43	43.48	54.81	42.85	38.53	54.35	56.36
1.5	1	20.37	25.29	20.71	16.49	24.17	32.32	13.52	20.39	13.14	11.48	19.84	23.37
1.75	1	9.86	13.17	9.93	7.86	12.44	18.11	6.25	10.42	5.96	5.41	10.15	12.86
2	1	5.79	8.29	5.86	4.63	7.63	12.03	3.67	6.36	3.56	3.31	6.27	8.18
1.25	1.25	34.52	39.39	34.75	28.37	37.55	27.89	24.48	31.93	23.70	21.18	31.33	19.69
1.25	1.5	16.59	20.61	16.81	13.48	19.05	14.51	11.07	16.28	10.83	9.46	15.98	9.95
1.5	1.5	11.31	14.14	11.31	8.85	13.15	8.59	7.21	11.02	7.12	6.15	10.55	5.81
1.5	1.75	7.14	9.33	7.15	5.64	8.87	5.65	4.50	7.24	4.36	3.97	7.29	3.99
1.75	2	3.95	5.44	3.95	3.17	5.07	3.19	2.57	4.21	2.53	2.35	4.24	2.34
2	2	3.23	4.44	3.12	2.62	4.13	2.58	2.14	3.53	2.08	2.01	3.43	1.98



(a) Shift in one quality variable (i.e. $a_2 = 1$) for the bivariate Gamma process



(b) Shift in both quality variables (a_1, a_2) for the bivariate Gamma process

Figure 5.5: *ARL* curves for the bivariate *MDMAX* dispersion chart with shift in one and both quality variables when *n*=5, $\rho = 0$ and $ARL_0 = 200$

Effect of α and β on the charts

Table 5.5 displays the *ARL* values for different bivariate Gamma distributed processes whist Figure 5.5 displays the *ARL* curves for the bivariate *MDMAX* dispersion chart for shifts in one and both quality characteristics. In Table 5.5, it is observed that the effect of β on the performance of the charts is not much significant (e.g. Gamma (2, 1) and Gamma (2, 2) have similar *ARL* values). It is also noted in Table 5.5 and Figure 5.5 that as α increases, the ARL_1 values for the different bivariate dispersion charts decreases.

5.6. Illustrative Examples

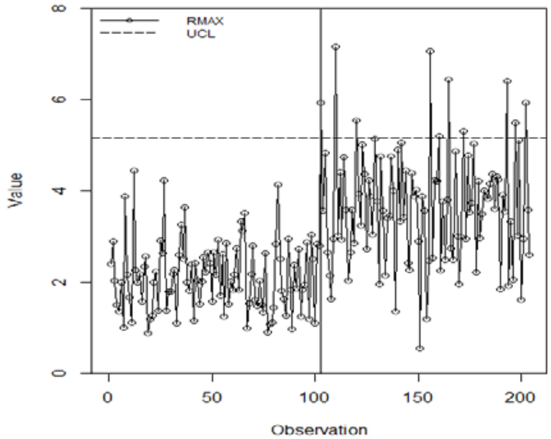
In these examples we illustrate how the charts perform with respect to monitoring shifts in the covariance matrix using the bivariate normal and *t* distributed process observations.

(a) Example 1: Real life data (Bivariate normal process)

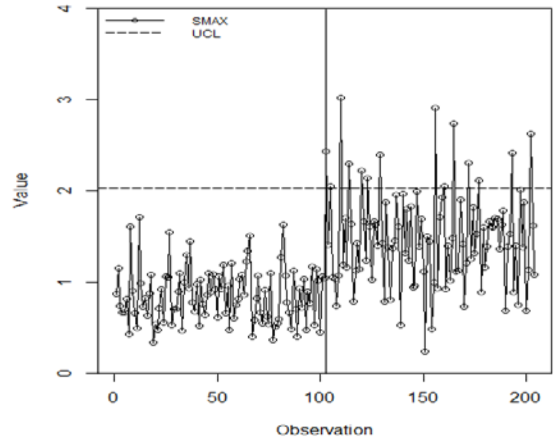
We use the dataset from the non- isothermal continuous stirred tank chemical reactor model given in Marlin (2000). The dataset consists of nine quality characteristics of 1024 observations and is used in the works of Yoon and MacGregor (2001), Shi *et al*. (2013) and Ahmad *et al*. (2014). We consider two of the variables which are correlated for this demonstration. They are inlet concentration of solvent flow (*X*) in $\mathrm{kmole/m^3}$ and flow rate of the solvent (*Y*) in $\mathrm{m^3/min}$ .The observations are standardized. The Royston (1983) multivariate test revealed that the bivariate dataset is normally distributed. We form 204 subgroups of size *n*=5 from the observations for each variable. We consider the target mean vector, covariance matrix (Σ_0) and correlation matrix (*r*) as;

$$\underline{\mu} = \begin{pmatrix} 0.103 \\ 0.9 \end{pmatrix},\ \Sigma_0 = \begin{bmatrix} 0.0026 & 0.0007 \\ 0.0007 & 0.0140 \end{bmatrix} \text{ and } r = \begin{bmatrix} 1 & 0.1238 \\ 0.1238 & 1 \end{bmatrix}$$

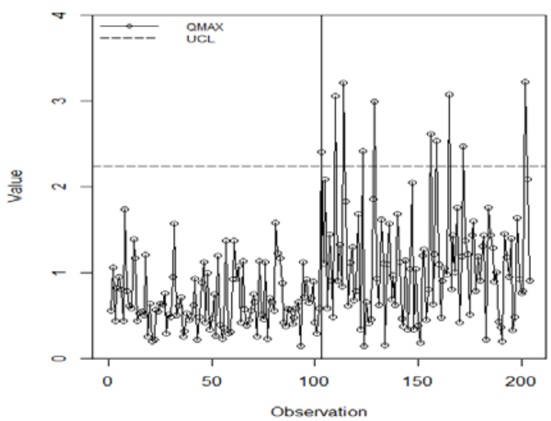
The first 510 observations forming the initial 102 subgroups are used to represent the in-control state but to monitor the process variability we induced a shift of magnitude $a_1 = 2$ into the second half of *X*. All the charts are calibrated to have an in-control *ARL* equals 200. The *UCL* values are decision values which are chosen for each chart to yield the required in-control *ARL* value. Using the known mean vector and covariance matrix, *UCL* was estimated in a simulation process which was repeated 10,000 times to give $ARL_0 = 200$ for *n*=5, considering observations from the bivariate normal process. The *UCL* for the *RMAX*, *QMAX*, *SMAX*, *MDMAX*, *MADMAX* and $|S|$ charts are determined as 5.154, 2.241, 2.029, 1.477, 2.57 and 5.29 respectively. The statistics for the various charts are displayed in Table 5.6. We have provided the first 40 of the 204 charting statistics for each chart due to limited space. Table 5.7 displays the out-of-control points in the charts whilst Figure 5.6(a)-(f) displays the different control charts for the dataset. We observe in Figure 5.6 that after the 102^{nd} sample, 14, 12, 10, 10 and 5 out-of-control points are detected in the *SMAX*, *MDMAX*, *RMAX*, *QMAX* and *MADMAX* charts respectively. The $|S|$ chart does not signal any out-of-control point. It is clear that for the set of observations used and the shift considered in this demonstration, the *SMAX* chart performed better than the other charts. The conclusions for this example are consistent with the results in Table 5.1.



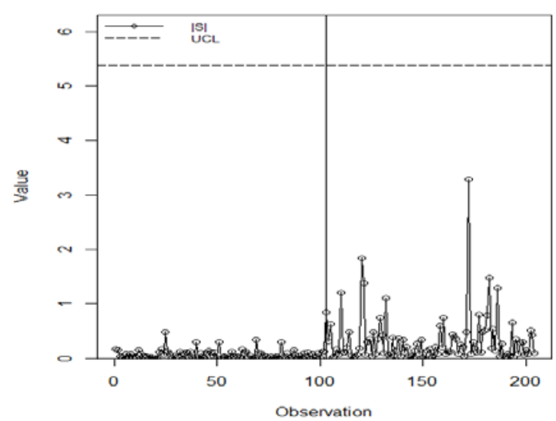
(a) *RMAX* chart



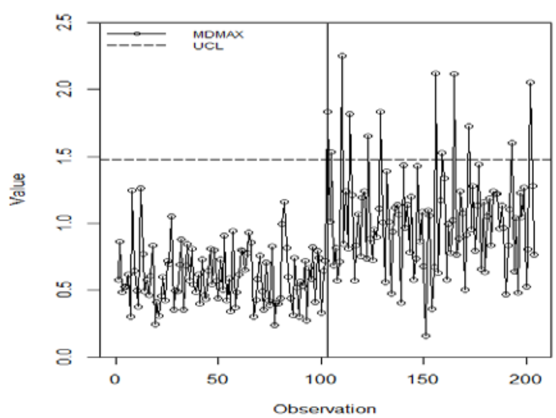
(b) *SMAX* chart



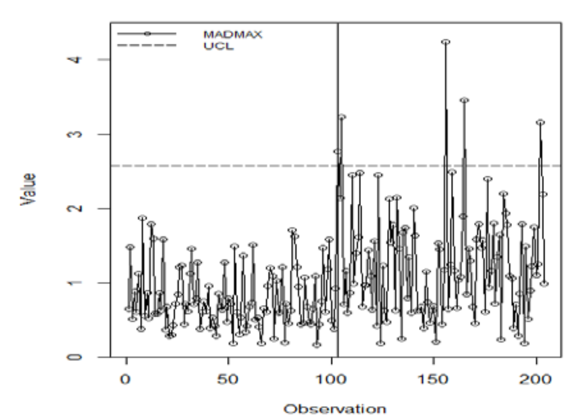
(c) *QMAX* chart



(d) $|S|$ chart



(e) *MDMAX* chart



(f) *MADMAX* chart

Figure 5.6: Bivariate dispersion control charts for example 1

Table 5.6: Data for illustration in example 1(first 40 out of 204 statistics)

Observations	*MDMAX*	|*S*|	*QMAX*	*MADMAX*	*RMAX*	*SMAX*
1	0.574	0.167	0.556	0.642	2.396	0.869
2	0.864	0.158	1.056	1.479	2.896	1.147
3	0.486	0.040	0.434	0.513	2.035	0.738
4	0.523	0.022	0.825	0.886	1.500	0.664
5	0.524	0.014	0.946	0.608	1.345	0.665
6	0.616	0.083	0.807	1.116	1.990	0.811
7	0.301	0.024	0.433	0.378	1.008	0.424
8	1.244	0.077	1.738	1.875	3.874	1.612
9	0.644	0.020	0.782	0.549	2.165	0.907
10	0.496	0.035	0.606	0.862	1.663	0.660
11	0.374	0.023	0.580	0.525	1.109	0.492
12	1.263	0.157	1.390	1.794	4.438	1.707
13	0.769	0.087	1.174	1.589	2.264	0.990
14	0.489	0.040	0.439	0.580	1.983	0.727
15	0.576	0.019	0.527	0.583	2.170	0.821
16	0.461	0.036	0.544	0.863	1.572	0.628
17	0.603	0.008	0.497	0.620	2.343	0.881
18	0.835	0.020	1.204	1.583	2.553	1.075
19	0.242	0.008	0.255	0.367	0.867	0.332
20	0.410	0.009	0.635	0.673	1.192	0.534
21	0.306	0.011	0.198	0.289	1.264	0.474
22	0.455	0.117	0.216	0.291	1.982	0.710
23	0.597	0.167	0.563	0.425	2.224	0.922
24	0.422	0.067	0.547	0.721	1.372	0.557
25	0.722	0.480	0.640	0.846	2.901	1.058
26	0.695	0.120	0.632	1.215	2.625	1.055
27	1.050	0.080	0.757	1.235	4.230	1.547
28	0.352	0.010	0.291	0.435	1.367	0.530
29	0.497	0.047	0.512	0.718	1.792	0.700
30	0.490	0.016	0.486	0.620	1.793	0.704
31	0.691	0.035	0.950	1.124	2.175	0.896
32	0.877	0.127	1.572	1.462	2.265	1.099
33	0.354	0.001	0.505	0.706	1.090	0.461
34	0.684	0.077	0.606	0.787	2.601	0.996
35	0.845	0.113	0.712	1.273	3.264	1.296
36	0.571	0.026	0.253	0.375	2.512	0.928
37	0.814	0.121	0.319	0.603	3.638	1.440
38	0.540	0.009	0.528	0.745	1.989	0.774
39	0.483	0.026	0.445	0.615	1.814	0.678
40	0.614	0.303	0.495	0.951	2.401	0.977

Table 5.7: Out-of-control points in the different bivariate dispersion charts

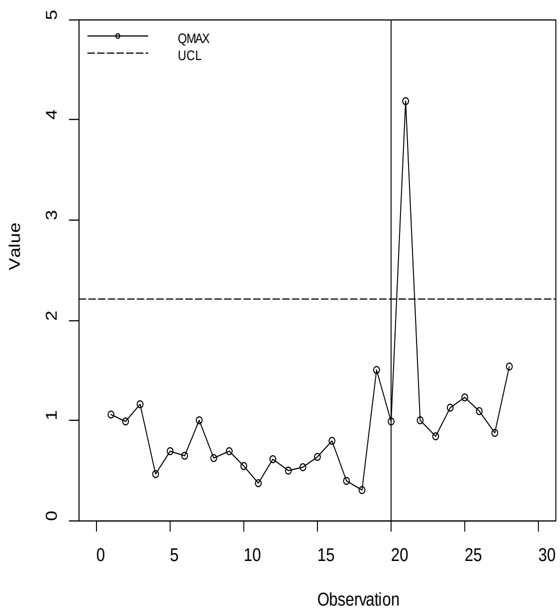
Charts	Out-of-control points
RMAX	103, 110, 120 ,156, 160, 165, 172, 193, 197 ,202
SMAX	103, 105, 110 ,114 ,120 ,123, 129 ,156, 160, 165, 172, 177, 193, 202
QMAX	103, 110, 114, 123, 129 ,156, 159, 165, 172, 202
$\|S\|$	0
MDMAX	103, 105, 110, 114 ,123, 129, 156, 159, 165 ,172 ,193, 202
MADMAX	103, 105, 156 ,165, 202

(b) Example 2: Simulated dataset (Multivariate t process)

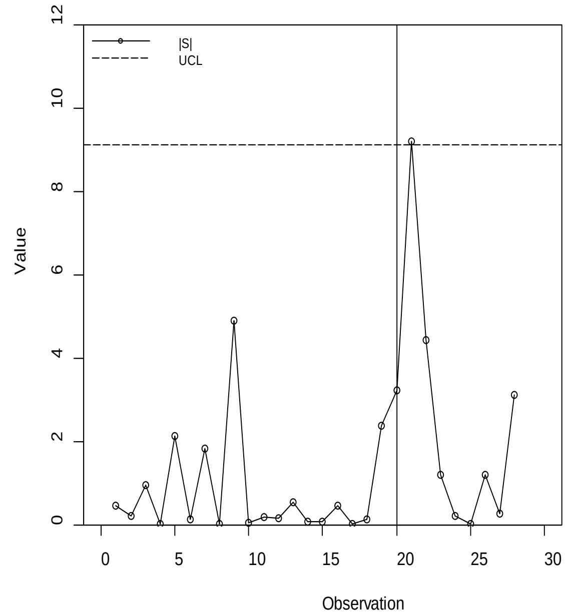
In this example, we simulated a dataset of 20 samples of size n=5 for each quality characteristic such that the dataset is bivariate t distributed with the following in-control parameters;

$$\underline{\mu} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \ \Sigma_0 = \begin{bmatrix} 1 & 0.3 \\ 0.3 & 1 \end{bmatrix} \text{ and } v=5$$

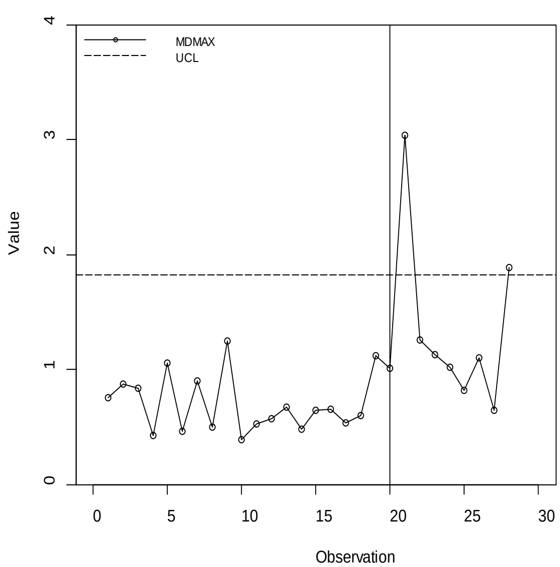
We generate an additional 8 samples of size n=5 by introducing a shift $a_1 = 2$ in the first quality variable. All the charts are calibrated to have an in-control *ARL*=200. The *UCL* for the charts can be seen in Table 5.2. Figure 5.7(a)-(d) displays the control charts for the dataset. We have omitted the graphs for the *RMAX* and *SMAX* charts from Figure 5.7. It is evident from Figure 5.7 that after the 20th sample, the *MDMAX* chart detects 2 out-of-control points and performed better than any other chart. The *QMAX*, *MADMAX*, *RMAX*, *SMAX* and $|S|$ charts signalled only 1 out-of-control point. Again the conclusions here are consistent with the simulation results from Table 5.2.



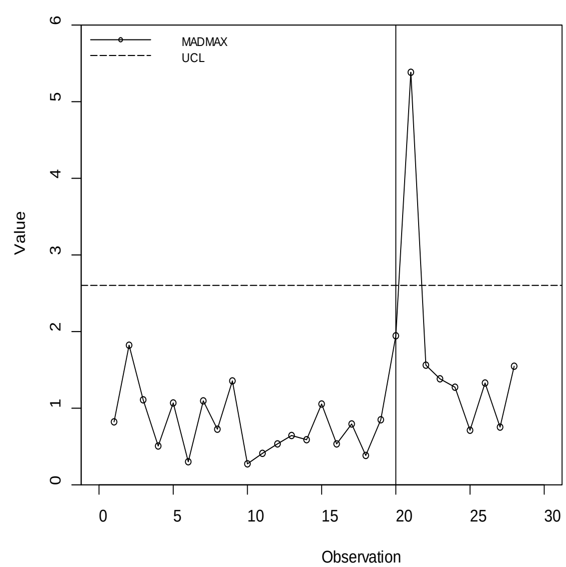
(a) *QMAX* chart



(b) $|S|$ chart



(c) *MDMAX* chart



(d) *MADMAX* chart

Figure 5.7: Bivariate dispersion charts for example 2

5.7. Conclusions

In the chapter, we presented bivariate charts based on the standard deviation (*S*), interquartile range (*Q*), average absolute deviation from median (*MD*), and median absolute deviation (*MAD*) statistics for dispersion monitoring of observations obtained from the bivariate normal and non-normal distributions. The performance of these charts is assessed and compared with the *RMAX* and $|S|$ charts using the *ARL* measure. For a bivariate normal process, it is evident that the *SMAX* chart, usually followed by the *MDMAX* and *RMAX* chart is superior to the other charts for monitoring assignable causes that affect one quality variable of a process. However, the *SMAX* and $|S|$charts slightly perform better for special causes that affect both quality variables for the bivariate normal process. Generally the *MDMAX* chart is noticed to have superior performance ability for medium to large shifts that affect one quality variable in the bivariate non-normal processes (*t*(5) and Gamma). For shifts in both quality variables, the $|S|$ chart usually performs better in the bivariate *t* distributed process (when *v*=5) whilst the *MDMAX* and $|S|$ chart performs slightly better than the other charts for the bivariate Gamma distributed process when the sample size is small.

The concentration of this study relied on observations from the bivariate process but the charts may be extended to processes with more than 2 quality characteristics.

CHAPTER 6

CONCLUSIONS

In the production process we strive to produce goods or outputs that meet the specifications outlined with minimum or no variation. To achieve this quality, statistical process control tools such as the control charts are used. The chart is used to monitor the process, detect abnormalities in form of shifts in the location and dispersion parameters promptly for corrective measures to be implemented to get the process back on track. This thesis concentrated on the monitoring of serially correlated and non-normally distributed processes. The outcomes from the work are summarised as follows;

Chapter 2 and 3 of the work discussed the application of residual and modified charts on autocorrelated data respectively. The residual charts were implemented by using the uncorrelated residuals obtained from the time series model fitted to the correlated data whilst the modified charts operated by applying the standard control charts on the correlated data with the control limits of the chart adjusted. The MEC and MCE residual and modified charts were proposed for monitoring autocorrelated data. In a performance comparison for autocorrelated data that can be modelled with an AR (1) process model, the MCE residual chart performed better than the Shewhart, CUSUM, EWMA, CSE and CSC residual charts for various shift and autocorrelation levels whilst the MEC residual chart slightly performed better than the existing charts for positively correlated processes in detecting small mean shifts. A similar conclusion is observed for the MCE and MEC modified charts discussed in Chapter 3.

Chapter 4 investigated the designs of the modified EWMA and CUSUM charts for serially correlated observations when the shift is unknown or uncertain. Faced with the challenge of an unknown shift value in the construction of the charts, the optimal parameters that minimize the *EQL* and *PCI* can be estimated and used in the charts. The loss due to poor quality is reduced by the minimization of the *EQL* value and the parameters (i.e. optimal) that possess the smallest *EQL* have a better detection ability in the range of shift for the autocorrelation level. The optimal parameters provided give the engineer an idea about the choice of parameters to use in process monitoring.

Chapter 5 focused on the use of bivariate control charts for monitoring shifts in the covariance matrix of bivariate *t* and Gamma distributed process. The charts named as *SMAX*, *QMAX*, *MDMAX* and *MADMAX* charts relied on dispersion estimates such as the standardized sample standard deviation (*S*), interquartile range (*Q*), average absolute deviation from median (*MD*) and median absolute deviation (*MAD*) respectively. In a comparison of these charts with the *RMAX* and $|S|$ charts for the bivariate non-normal parent distributions (*t*(5) and Gamma), it was shown that the *MDMAX* chart usually performed better than the other charts for medium to large shifts in one quality characteristic of the process. The investigations also revealed that with shifts in both quality characteristics, the $|S|$ chart in the bivariate *t*(5) distributed process and the *MDMAX* and $|S|$ charts in the bivariate Gamma distributed process were superior to their counterparts when the sample size was small.

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APPENDIX

Table A1: *ARL* values for the bivariate charts when n=10 and $ARL_0 = 200$ for the bivariate normal process

		RMAX			*QMAX*			*SMAX*			*MDMAX*			*MADMAX*			|S|		
	ρ	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
	UCL	5.664	5.664	5.644	1.899	1.898	1.895	1.683	1.682	1.678	1.3298	1.3296	1.325	2.042	2.04	2.037	3.625	3.298	1.8483
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	203.90	199.47	198.36	198.41	197.38	200.29	202.72	201.73	198.86	199.95	198.90	199.00	202.04	199.63	202.09	200.82	199.66	200.94
1.25	1	21.59	21.46	21.13	30.67	30.64	30.59	16.00	15.73	15.63	19.45	19.43	19.17	33.66	33.01	32.70	26.59	27.25	27.05
1.5	1	5.36	5.31	5.31	8.88	8.88	8.85	3.98	3.90	3.90	4.66	4.66	4.65	9.86	9.81	9.76	8.47	8.36	8.49
1.75	1	2.53	2.51	2.48	4.28	4.20	4.20	1.99	1.98	1.96	2.30	2.30	2.26	4.64	4.62	4.60	4.20	4.12	4.11
2	1	1.67	1.67	1.66	2.71	2.70	2.69	1.42	1.40	1.40	1.59	1.59	1.57	2.85	2.84	2.83	2.68	2.65	2.62
1.25	1.25	11.69	11.72	12.18	16.87	16.94	17.75	8.43	8.65	9.30	10.29	10.59	11.06	17.97	18.39	19.35	6.88	6.89	7.00
1.25	1.5	4.51	4.56	4.71	7.28	7.32	7.56	3.34	3.35	3.56	4.03	4.08	4.28	7.97	8.15	8.21	3.21	3.25	3.26
1.5	1.5	2.95	3.02	3.30	4.78	4.82	5.14	2.26	2.34	2.57	2.65	2.75	2.97	5.29	5.32	5.55	1.97	1.94	1.96
1.5	1.75	1.95	2.02	2.16	3.19	3.23	3.31	1.58	1.64	1.72	1.81	1.84	2.00	3.37	3.40	3.63	1.48	1.50	1.49
1.75	2	1.32	1.34	1.44	1.94	1.96	2.06	1.18	1.19	1.23	1.26	1.29	1.36	2.05	2.07	2.16	1.14	1.14	1.14
2	2	1.19	1.20	1.28	1.66	1.69	1.77	1.10	1.11	1.14	1.16	1.17	1.24	1.74	1.76	1.85	1.07	1.07	1.08

Table A2: *ARL* values for the bivariate charts when n=10 and $ARL_0 = 200$ for the bivariate $t(5)$ process

		RMAX			QMAX			SMAX			MDMAX			MADMAX			|S|		
	ρ	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
	UCL	8.86	8.819	8.675	1.768	1.765	1.759	2.47	2.45	2.41	1.489	1.484	1.475	1.902	1.9015	1.896	7.78	7.02	3.94
a1	a2	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL	ARL
1	1	202.84	201.38	201.77	200.65	199.90	201.81	201.64	200.23	200.90	199.72	203.09	200.12	200.09	202.15	199.52	200.70	202.67	201.11
1.25	1	83.36	82.86	81.63	42.40	41.98	41.49	81.86	78.43	77.00	44.12	43.81	43.05	43.42	43.35	43.15	73.28	72.63	72.49
1.5	1	34.54	33.51	32.89	12.73	12.58	12.56	31.06	30.14	27.50	12.33	12.07	11.78	13.63	13.43	13.28	32.93	32.67	32.26
1.75	1	15.84	15.50	14.49	5.89	5.84	5.74	12.92	12.59	11.53	5.08	5.05	4.76	6.25	6.21	6.15	17.87	17.60	17.42
2	1	8.59	8.42	7.87	3.51	3.48	3.43	6.45	6.25	5.84	2.84	2.85	2.78	3.75	3.72	3.70	10.78	10.67	10.60
1.25	1.25	54.13	54.95	55.21	24.04	24.15	24.78	51.81	52.04	52.43	26.25	26.54	27.17	24.11	24.60	25.26	27.37	27.14	27.32
1.25	1.5	28.22	28.37	28.40	10.41	10.60	10.79	25.43	25.46	25.71	10.26	10.36	10.74	11.11	11.20	11.33	13.66	13.35	13.30
1.5	1.5	19.38	19.41	20.04	6.78	7.00	7.34	17.51	17.56	17.72	6.89	7.08	7.34	7.24	7.42	7.68	7.22	7.18	7.17
1.5	1.75	12.09	12.12	12.24	4.37	4.42	4.65	9.90	9.93	10.04	3.96	4.03	4.20	4.59	4.74	4.90	4.56	4.54	4.43
1.75	2	6.08	6.21	6.32	2.50	2.52	2.67	4.83	4.84	4.94	2.19	2.20	2.40	2.68	2.68	2.82	2.33	2.30	2.35
2	2	4.85	4.94	5.04	2.09	2.11	2.24	3.75	3.76	3.90	1.82	1.84	1.97	2.19	2.23	2.35	1.86	1.83	1.85

Table A3: ARL values for the bivariate charts when n=10 and $ARL_0 = 200$ for the bivariate Gamma (1, 1) process

		RMAX			QMAX			SMAX			MDMAX			MADMAX			|S|		
	ρ	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7	0	0.3	0.7
	UCL	8.193	8.189	8.05	2.04	2.038	2.03	2.5192	2.514	2.4875	1.534	1.532	1.521	1.86	1.859	1.854	8.48	7.795	4.325
a1	*a2*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*	*ARL*
1	1	201.91	200.08	200.72	200.92	200.96	201.53	199.93	201.06	200.37	200.60	199.63	200.58	200.62	200.32	200.35	200.31	200.71	200.78
1.25	1	65.21	64.92	63.82	56.72	56.60	56.20	62.45	61.15	59.88	44.12	43.93	43.40	56.91	57.23	57.10	66.29	65.44	64.51
1.5	1	25.02	24.95	23.94	20.10	19.96	19.22	21.72	21.59	20.95	13.44	13.41	13.11	20.24	20.19	19.76	30.46	30.02	29.13
1.75	1	12.11	12.05	11.51	9.61	9.58	9.56	10.01	10.09	9.84	6.19	6.11	6.07	9.79	9.75	9.72	16.96	16.68	16.46
2	1	7.08	7.03	6.70	5.83	5.77	5.70	5.92	5.71	5.69	3.67	3.63	3.62	5.85	5.82	5.80	10.50	10.72	10.39
1.25	1.25	40.20	40.40	41.52	32.93	33.75	34.79	36.76	37.22	38.47	24.97	25.59	26.80	32.78	33.27	35.10	25.73	25.31	24.50
1.25	1.5	20.02	20.53	20.69	16.31	16.52	17.10	17.99	17.97	18.79	11.06	11.55	11.79	16.13	16.50	16.72	13.09	13.09	12.98
1.5	1.5	13.60	14.12	14.70	10.60	10.95	11.48	11.87	12.14	13.14	7.23	7.48	8.24	10.79	11.12	11.55	7.52	7.49	7.47
1.5	1.75	8.80	8.99	9.33	6.92	7.05	7.58	7.50	7.59	8.11	4.55	4.74	5.27	6.90	7.19	7.51	4.97	5.07	4.90
1.75	2	4.79	4.90	5.30	3.95	4.04	4.29	4.03	4.20	4.56	2.61	2.69	3.01	3.96	3.97	4.33	2.72	2.79	2.75
2	2	3.83	4.08	4.33	3.17	3.26	3.49	3.23	3.40	3.73	2.14	2.24	2.52	3.16	3.30	3.55	2.18	2.24	2.23

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